The TREND IN ENGINEERING

at the University of Washington

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1) It stimulates, promotes, and coordinates investigations and research in all fields of engineering.
2) It publishes results of significant research achievements.
3) It provides opportunities through Graduate Research Assistantships for engineering students to extend their professional education while pursuing a course of study leading to the M.S. or Ph.D. degree.

For administrative purposes the work of the Station is organized under six divisions. All research is carried on in the various academic departments of the College of Engineering under the supervision of the department faculties.

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UNKNOWING DUPLICATION IN RESEARCH— A PERENNIAL TRAGEDY*

SENATOR HUBERT H. HUMPHREY, *Chairman, Subcommittee on Reorganization and International Organizations, U. S. Senate Committee on Government Operations*

"How much research and development needlessly duplicates prior work?"

This is a crucial question, and no one is sure of the answer. Some estimates are that, in Federally sponsored research and development, unknowing duplication ranges from 10 per cent to as high (in developmental work) as 50 per cent.

Whatever the level, it represents a tragic, pitiful waste in men, money, and material: our nation cannot afford this waste. Reliable information systems must be established and fully utilized in order to reduce the degree of unknowing duplication to an absolute minimum.

Technical writers and publishers realize this problem because they encounter it almost every day. They are aware of a critical breakdown in the nation's scientific communications despite, or often because of, "mountains" of technical literature. Consequently writers and publishers often go to considerable effort, prior to writing or releasing a "new" research-and-development report to try to determine whether it is genuinely new or has been recorded previously elsewhere. Despite such efforts, present information systems are so inadequate that so-called "new" reports will appear in print but will merely turn out to be repetitions of knowledge buried elsewhere.

The consequence is frustration and disappointment, not only to the writer and the publisher, but to the scientist or engineer whose work is being described. A man or team sometimes spends months or years on work that might have been avoided if information on the real "state-of-the-art" had been more accessible in the first place. For every instance of outright duplication there are probably scores of cases in which at least part of the painstakingly performed work could have been eliminated if obscure data had been more accessible at the outset.

Improved Information Systems

In view of these facts, the Senate Committee on Government Operations has been striving for years to improve Federal Agencies' information systems. There are strong fiscal and other reasons for our efforts: The Federal Government is spending $9 billion a year for scientific research, development, testing, and evaluation. Of that sum, three-fourths is for national defense. A rising proportion is for civilian space science. The over-all work involves over 160,000 projects now under way in over 9000 facilities. Many of these tasks overlap. No one can foretell which projects may be affected by work under way elsewhere. Projects in the life sciences interact with and draw upon projects in the physical sciences and vice versa. Today's physics, chemistry and biology converge on common targets and require interdisciplinary skills.

Time is the crucial dimension. Manpower—particularly the most qualified scientists, engineers, and technicians—is in scarce supply. A single man-year spent needlessly repeating experiments performed earlier is a man-year which can never be regained.

Managing Science Information

In order to help remedy the breakdown in communications, the Senate Committee on Government Operations has issued a series of publications on the subject of managing nonclassified science information. These publications include:

1. Senate Document 113, 86th Congress, "Documentation, Storage, and Retrieval of Information," an analysis of the problem of managing *published* literature.

2. Senate Report 243, 87th Congress, "Coordination of Information on Current Scientific Research and Development Supported by the United States Government," which analyzes the problem of managing *pre*-publication literature.

The latter report has opened up a relatively new vista in the area of science information. For years, scientists, engineers, writers and publishers have worried about the problem of how to find articles or reports which are already in print. They have sought ways to find an article, monograph, or book in its original form or in abstract, index, or other secondary form.

Now, however, our latest Senate report reveals another critical problem: how to locate knowledge about research and development that is still in progress. This work is not recorded anywhere and it may never be recorded. Some of this work may be reported orally at a professional meeting, but it may never be published; indeed some work may not even be orally communicated.

Why? Perhaps because it ends in "failure," or because a Federal contract or subcontract is cancelled, or a key person leaves and the project dies or is altered. Maybe the project is completed, but the scientist or engineer turns so quickly to other fields that he doesn't bother to write down the results. Perhaps the recipient "buries" the report instead of disseminating it.

Unjustifiable Duplication

Whatever the reason, vast amounts of information sink disastrously without trace. What is the result? Someone somewhere unknowingly reperforms the same or similar work. Such unknowing duplication (in contrast to planned, intentional duplication) cannot be justified.

What then is the answer? A bold, imaginative program must be launched to strengthen the nation's largely archaic science information systems. The latest electronic data processing methods—already in operation in some of the systems—must be refined and utilized. This program requires the fullest cooperation of private industry; professional societies; Federal Agencies; and nonprofit institutions, universities, and laboratories.

One element of the program should be vigorous support of what is called the Science Information Exchange (SIE), which was established in September 1960 by a number of Federal Agencies with the Smithsonian Institute. This organization is designed to register and index 200-word

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* This editorial is reproduced from the STWP *Review*, Vol. 8, No. 3, July, 1961, p. 2 ff. The basic problem of communication it discusses was treated editorially in *Trend* for last October; but Senator Humphrey uncovers new aspects of the subject, and the problem deserves emphasis as indicated by the numerous words the professional journals have given it during the past year. A noteworthy example is the special issue of the *Research Engineer* (Georgia Tech EES) for last April given entirely to "The Information Problem."

summaries of current projects supported by the U. S. Government in the physical, social, engineering, and mathematical sciences. Since 1953, there has existed a comparable and reasonably successful Federal Exchange, in the biosciences. It registers 30,000 current medical, biological, and related projects. Through BSIE, as it was formerly known, any medical scientist anywhere in the United States—or anywhere in the world—can find out who may be currently engaged in particular research; or he may trace research done one, two, three, or four years ago which was never reported in the technical literature, but which may show up on the index of terminated projects.

Extension to Physical Sciences

The task now is to secure voluntary registration of abstracts of current research-and-development work in the physical sciences.

When it becomes operational in the new fields of its activity, the SIE can be an invaluable tool for Federal administrators, scientists, and engineers. Later, after Federal needs are begun to be fulfilled, non-Federal sources should have access to SIE including, of course, writers, editors, and publishers.

SIE is a key to knowledge about the very existence of a project. Ideally, however, it should fit in with documentation centers that contain copies of reports generated by the project. Such integration is not an easy task, but it is far from insoluble.

There are no less than seven Federal documentation systems today: (1) the Library of Congress; (2) the U. S. Department of Agriculture Library; (3) the National Library of Medicine; (4) the Armed Services Technical Information Agency; (5) the Office of Technical Services, Department of Commerce; (6) the Atomic Energy Commission's Information Division; and (7) the National Aeronautics and Space Administration's Technical Information Division.

In addition, there are vast numbers of Material Information Centers containing Federally sponsored reports on a vast variety of metals, minerals, etc.

No single center is or can be complete unto itself. So many of them overlap in coverage that it would be tragic for them to go their separate ways. There is every reason to secure optimal voluntary cooperation between all of these various centers and systems. Together they can serve their combined clienteles with bibliographic and other services which could greatly accelerate the nation's technical effort. Together they can help achieve a level of efficiency which will help reduce the inexcusable waste caused by unknowing partial or complete duplication.

Complete Exchange Required

A national science information network is needed. It cannot and must not be a monolith like the Soviet Union's system. Democracy need not and should not imitate totalitariansim, but neither should it fumble or blunder.

The breakdown in scientific communications must be remedied. The communications arteries, blocked with "fat," must be cleared. Scientists, engineers, and technicians must be enabled to obtain nonclassified data when they want it and in the way they want and need it.

The alternative is to come out "second best" in the scientific race. This must not and will not happen.

If you are interested in making sure of U. S. success in science information, the time has come for you to act. Your professional society can be, as it has been, your spokesman for progress in science information.

And the Search Continues*

Every 24 hours enough technical papers are produced around the world to fill seven sets of the *Encyclopedia Britannica*. One man, scanning the 60 million pages of technical literature produced last year would need 465 years to finish the job.

The worst is yet to come. The total quantity of technical literature in existence doubles every ten years. Scientific expenditures are going up even faster, increasing at 10 per cent annually, which will eventually bring even more emphasis to scientific publication...

Recent developments in computer processing, photographic recording, and other techniques have glamorized the information field. The technical documentalist is provided with exotic tools interesting to scientists and laymen alike.

The most impressive equipment combines computers with very small pictures of the original documents. The photographic copies (or *storage*) permit the filing of collections of information in very small volumes. Specialized libraries can be recorded in boxes which only line part of a wall in a medium-sized room. Some of these machines can select references which contain desired data and produce a full size copy.

Difficulties still exist. The biggest bottle-necks in information processing are found in indexing, abstracting, and translating. It's hard to think of a machine studying an article and preparing an abstract. Yet, fairly reasonable abstracts are being prepared by machine already, and some progress has been made with machine indexing. Translation by machine is receiving a great deal of attention, and useful results will be economically feasible within a few years...

* From L. W. Ross and R. J. Kyle in the *Research Engineer*, Georgia Tech EES, April, 1961

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Modulated Traveling-Wave Antennas*

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D. K. Reynolds

Demands for increased performance in communications systems, radar, and radio astronomy have in recent years made necessary antenna systems of ever-increasing size, complexity, and cost. As examples, a steerable parabolic-reflector antenna 600 ft in diameter is now under construction by the Navy in West Virginia; a linear antenna array 5400 ft long has recently been completed, also for the Navy, in Texas; and a fixed parabolic reflector antenna over 1000 ft in diameter is under construction in Puerto Rico. This article describes results of a research program in the properties of antennas of potentially very large size, carried out in the Department of Electrical Engineering under sponsorship of the Office of Naval Research.

Aperture vs End-Fire Antennas

The directivity of an antenna measures the ratio of the radiated power density in a given direction to that which would be produced if the same total power were radiated equally in all directions. Antennas of high directivity are invariably one of two types, either the aperture (broadside) or the end-fire. As has long been recognized, axial length may be traded for aperture area in the attainment of high directivity. A square aperture of length and width d, illuminated with an aperture efficiency of 60%, has a directivity

$$D \cong 7.5 \left(\frac{d}{\lambda}\right)^2, \qquad (1)$$

where λ is the free-space wavelength. An end-fire aperture of length L, excited with uniform amplitude and with a uniform phase delay between successive radiators, adjusted to give maximum directivity, has a directivity

$$D \cong 7.5 \frac{L}{\lambda}. \qquad (2)$$

Inspection of Eq 1 and 2 shows that for extremely high directivity, such as 10,000 or more, the aperture antenna has a much smaller maximum dimension than the end-fire. However, for moderate directives such as 100 or thereabouts, the difference in dimensions between the two types is not so great, and economic considerations may well enter into the choice between the two. For example, equal directivity (114, or 20.6 db over an isotropic source) can be obtained with either a 100-ft end-fire antenna or an aperture antenna 25.5 ft on a side, at a frequency of 150 Mc. For the important application of fixed, point-to-point communications, the longer end-fire antenna can be mounted on simple posts; and it may well be considerably less expensive than the corresponding aperture antenna.

One of the most common forms of the end-fire antenna is the type excited by means of a traveling wave guided along the antenna structure. Traveling-wave antennas of the Yagi type consisting of arrays of parallel rods or discs have been successfully made in lengths up to about 15 wavelengths. [1, 2, 3, 4] At one time there was thought to be a practical upper limit to the length, and hence the directivity, of a traveling-wave antenna. This restriction was removed by Simon,[5] in 1954, with the development of the "modulated" traveling-wave antenna, in which the phase velocity of the wave guided by the array is cyclically increased and decreased, by appropriate variations of the array parameters. The cycles of "modulation" act as individual arrays, whose resultant field may be made to add in phase along the axis of the array. While such design theoretically removes the restriction on maximum length of the array, other limitations appear that seriously affect the performance of the antenna. These are discussed in the following sections, and a new design is presented that promises to overcome the major faults of the modulated array.

* The work described in this article was sponsored by the Office of Naval Research, U.S. Navy, under contract Nonr-3091 (01) (X). The author wishes to acknowledge the assistance of two graduate students in Electrical Engineering, Robert F. Tighe and William W. Cooley, who contributed to the work of the contract.

Radiation from Phase-Modulated Traveling-Wave Antennas

The field pattern of a traveling-wave antenna, assumed to be an array of closely spaced isotropic sources of total length L, excited with a phase shift of β_o radians per unit length, and with constant amplitude, is

$$E(\theta) \alpha \frac{\sin \frac{L}{2}(k \cos \theta - \beta_o)}{\frac{L}{2}(k \cos \theta - \beta_o)} \qquad (3)$$

where θ = angle with respect to the axis, $k = 2\pi/\lambda$, and λ = free-space wavelength.

Equation (3) closely approximates the field pattern of long end-fire dipole arrays with four or more elements per wavelength. As has long been recognized, the maximum directivity for such an antenna is obtained for

$$(\beta_o - k)\, L \cong \pi . \qquad (4)$$

Equation (4) is known as the Hansen-Woodyard condition. A uniform traveling-wave antenna is normally designed by choosing a length, L, to yield the desired gain as given by Eq (2) and then choosing β_o to satisfy Eq (4). The required value of β_o is usually obtained from experimentally determined curves of β_o vs the parameters of the array. A typical family of such curves, for arrays of parallel rods, is shown in Fig. 1.

From Eq (4) we see that, as L is made large, β_o must be only slightly larger than k. This requires that the elements of the array be operated so far from their resonant lengths that the array becomes difficult to excite. Any simple exciter such as a driven dipole and reflector at one end of the structure, will be loosely coupled to the array, and a major part of the radiation will come directly from the exciter.

The phase-modulated array overcomes this limitation in the following way: Consider the antenna again as an array of closely spaced isotropic sources, excited with constant amplitude, I_o, but with a relative phase shift with respect to the center of the array, given by

$$I(x) = I_o \epsilon^{-j(\beta_o x + m_p \sin \frac{2\pi x}{\lambda_m})} \qquad (5)$$

where x = distance along the array measured from the center, m_p = a constant (phase-modulation index), and λ_m = axial distance for one cycle of phase modulation.

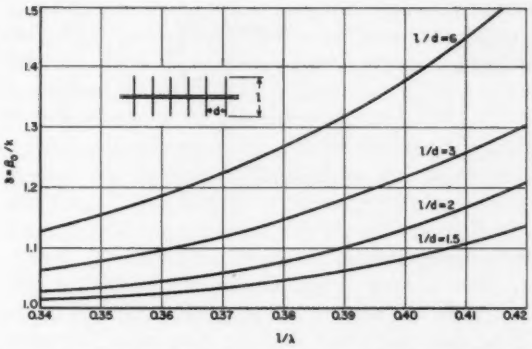

FIG. 1. PHASE CONSTANT OF WAVES LED ALONG ARRAYS OF PARALLEL RODS, AS FUNCTIONS OF THE PARAMETERS

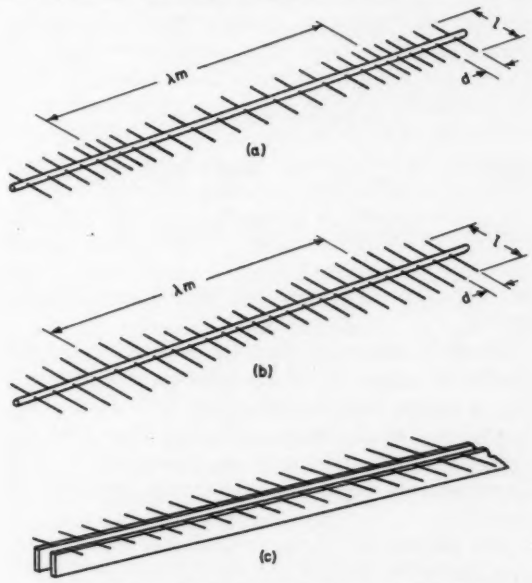

FIG. 2. MODULATED TRAVELING-WAVE ANTENNAS
(a) Density-modulated Yagi array
(b) Length-modulated Yagi array
(c) Driven array with amplitude modulation

Phase modulation of the traveling wave may be produced in antennas of the Yagi type (arrays of parallel rods) either by periodically varying the spacing between adjacent rods, as sketched in Fig. 2a, or by varying the lengths, as shown in Fig. 2b. If the cyclical variation is quite gradual, the design data shown in Fig. 1 can be used.

The radiation pattern resulting from Eq (5) is given by

$$E(\theta) \alpha J_o(m_p) \frac{\sin (U - \frac{\pi L}{\lambda_m})}{U - \frac{\pi L}{\lambda_m}} \qquad (6a)$$

$$+J_1(m_p)\frac{\sin\left(U-\frac{2\pi L}{\lambda_m}\right)}{U-\frac{2\pi L}{\lambda_m}} \tag{6b}$$

$$-J_1(m_p)\frac{\sin U}{U} \tag{6c}$$

$$+J_2(m_p)\frac{\sin\left(U-\frac{3\pi L}{\lambda_m}\right)}{U-\frac{3\pi L}{\lambda_m}} \tag{6d}$$

$$+J_2(m_p)\frac{\sin\left(U+\frac{\pi L}{\lambda_m}\right)}{U+\frac{\pi L}{\lambda_m}} \tag{6e}$$

+ higher order terms,

where J_0, J_1, J_2, etc. are Bessel Functions of the modulation index, m_p, $U=\frac{L}{2}\left[k\cos\theta-\left(\beta_o-\frac{2\pi}{\lambda_m}\right)\right]$, and the other quantities are defined as in Eq (3) and (5).

The field pattern is thus the sum of an infinite number of terms, all of the form $\sin x/x$, differing only in relative displacement along the U axis. If m_p, is relatively small (for example, less than 0.5), the terms in $J_3(m_p)$ and higher are negligibly small in comparison to the lower-order terms, and may be neglected.

The method of Simon consists in choosing λ_m such that

$$\beta_o-\frac{2\pi}{\lambda_m}=k\,. \tag{7}$$

With (7), term (6c) becomes

$$-J_1(m_p)\frac{\sin\frac{L}{2}k(\cos\theta-1)}{\frac{L}{2}k(\cos\theta-1)}\,. \tag{8}$$

Equation (8) is the field pattern of an end-fire antenna excited with uniform amplitude and having a progressive phase shift per unit length equal to that of a plane wave in free space. Under these conditions, terms (6a), (6b), and (6d) are equivalent to patterns of antennas excited with waves having phase velocity less than the free-space value, and term (6e) to one excited by a wave with phase velocity greater than free space. The maxima of the slow-wave patterns occur at "imaginary" angles, where the argument of the sine is greater than unity. These terms contribute largely to reactive energy near the antenna. The contribution of these terms at real angles, which chiefly affects the side-lobe level of the antenna, can be made small by proper choice of m_p, λ_m, and β_o. Term (6e) has its maximum at a real angle, this maximum appearing as a side lobe on the overall pattern.

The purpose of phase modulation is to allow a much larger value of the average phase constant, β_o, than is possible with the unmodulated array. Thus, even in very long arrays where the Hansen-Woodyard condition would require β_o to be almost equal to k for the unmodulated array, phase modulation allows a much larger value to be used, and the array becomes easy to excite with a concentrated exciter at one end.

The principal terms of Eq (6) are illustrated in Fig. 3a, for a particular case in which $m_p=0.4$. This kind of "wave spectrum" illustrates the formation of the radiation pattern from the component waves exciting the antenna. The term whose wave number is 1.0 is relied upon to provide the end-fire radiation. The large amplitude term at 1.1 contributes only to the side lobes at real angles off the antenna axis.

Optimum Design

Further examination of Eq (6) shows that with the proper choice of β_o, λ_m, and m_p the lobes of terms (6a) that occur at real angles may be made of opposite sign, and may have about the same amplitude as the side lobes of term (6c). Term (6a) has so little amplitude at real angles that it can be neglected in most cases. The principal maximum of term (6e) can not be cancelled entirely by the other terms, however, since it occurs on a lobe of width 2π, while all minor lobes are of width π.

For maximum cancellation between the minor lobes of terms (6c) and (6a), β_o must satisfy

$$[\beta_o-k]\frac{L}{2}=2n\pi\;;\;n=1,\,2,\,3,\,\ldots \tag{9}$$

Equations (7) and (9) now specify β_o and λ_m and they imply that there must be an integral number of cycles of modulation on the antenna. The integer n and the modulation index m_p are chosen on the basis of the best compromise between the completeness of side-lobe cancellation and amplitude of the maximum of term (6e).

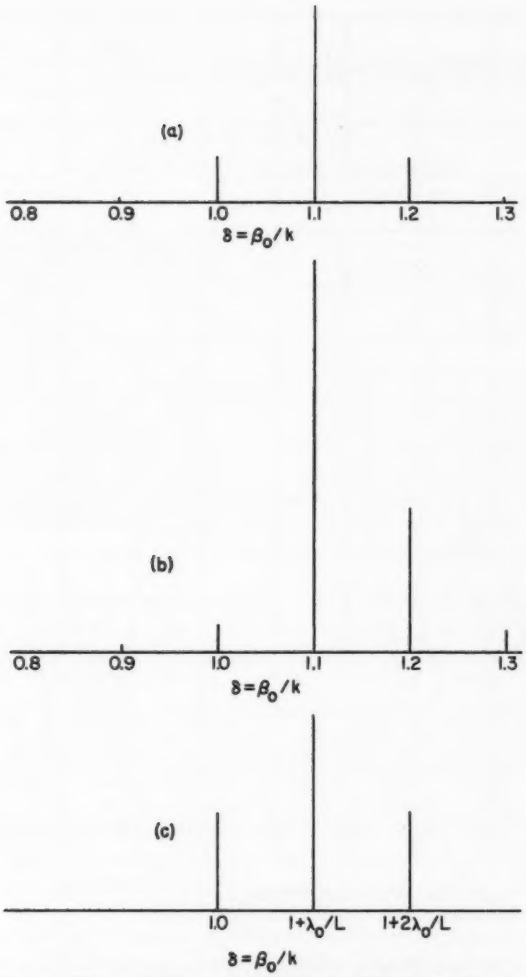

FIG. 3. WAVE SPECTRA OF TRAVELING-WAVE ANTENNAS
(a) Pure phase modulation
(b) Combined phase and amplitude modulation
(c) Pure amplitude modulation

This low-side lobe design is unfortunately only theoretical, since it demands an antenna in which the traveling wave is phase modulated at constant amplitude. Measurements of actual models, however, show that a strong modulation of the amplitude of the traveling wave inevitably accompanies the phase modulation. The effect on the wave spectrum is as shown in Fig. 3b, for a typical case. The amplitude of the component wave of wave number 1.0, which was relied upon to yield end-fire radiation, has been greatly reduced in comparison to the slow-wave component at 1.1. As a result, a low side-lobe design is not feasible; on the other hand, useful end-fire radiation can be obtained if the antenna parameters are so chosen that a minor maximum of the radiation from the high-amplitude term is adjusted to reinforce the main lobe of the term at 1.0. This requirement is satisfied if there are an odd number of half-cycles of modulation along the entire antenna.

An experimental antenna of 65 wavelengths total length with 6½ cycles of modulation, was constructed and tested. The lengths of the rods were made constant, and the number per unit length was cyclically varied. The resulting radiation pattern is shown in Fig. 4a. The main lobe is seen to have a width of about 8½ deg between half-power points. The first-order side lobes are quite high, being only about 6 db below the main-lobe amplitude; however, the directivity of the antenna is still very high.

While the long modulated traveling-wave antenna has thus been shown to be feasible, it has a very serious drawback. The frequency band over which the radiation pattern, and hence the directivity, is substantially constant is very small. This result is to be expected, since proper operation of the antenna relies on the correct phasing of the centers of radiation all along the axis. A variation in frequency from the design value causes a phase error to accumulate along the antenna. If the antenna is very long, the phase error per unit length must be very small in order not to accumulate sufficiently to ruin the radiation pattern. Analysis has shown that the useful frequency band of the antenna of Fig. 4a is only about 1.3%. For greater lengths, the bandwidth tends to be inversely proportional to the length. Because of the limitations of narrow bandwidth and high side lobes, an amplitude-modulated design was investigated, as described below.

Amplitude Modulation and Driven Arrays

If the antenna excitation is of the form

$$I(x) = I_o(1+m_a \cos\frac{2\pi x}{\lambda_m})\, \epsilon^{-j\beta_o x} \qquad (10)$$

(amplitude modulation at constant phase velocity), the resulting pattern is

$$E(\theta) \propto \frac{\sin\,(U-\frac{\pi L}{\lambda_m})}{U-\frac{\pi L}{\lambda_m}} \qquad (11a)$$

$$+\frac{m_a}{2}\,\frac{\sin U}{U} \qquad (11b)$$

$$+\frac{m_a}{2}\,\frac{\sin\,(U-\frac{2\pi L}{\lambda_m})}{U-\frac{2\pi L}{\lambda_m}} \qquad (11c)$$

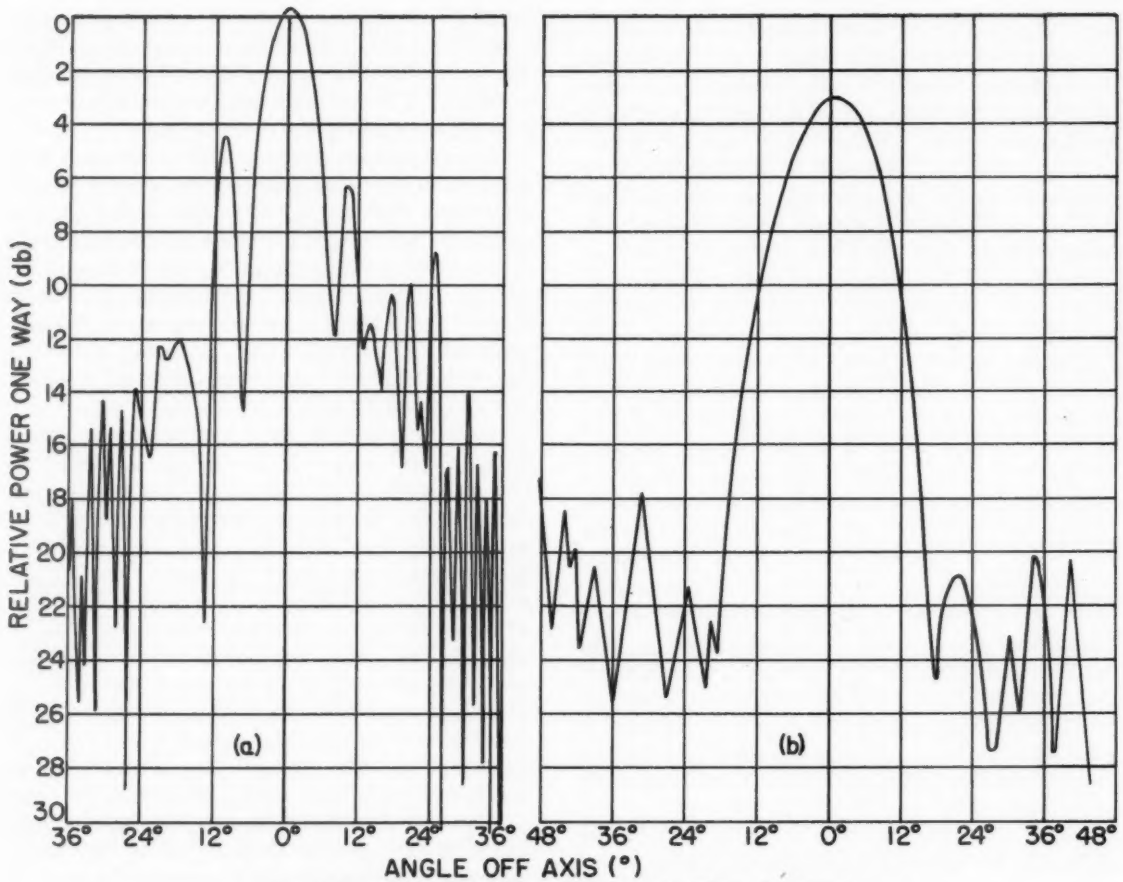

FIG. 4. RADIATION PATTERNS OF TRAVELING-WAVE ANTENNAS
(a) 65-wavelength modulated Yagi antenna; (b) 15-wavelength driven traveling-wave antenna

where U is defined as before. Again, if β_o and λ_m are chosen such that

$$\beta_o - \frac{2\pi}{\lambda_m} = k \tag{12}$$

and

$$(\beta_o - k)\frac{L}{2} = (2n-1)\pi \text{ and } n = 1, 2, 3, \ldots \tag{13}$$

an end-fire beam with low side lobes results.

A most interesting case occurs for $n=1$, $m_a=1$. Then

$$\lambda_m = L \tag{14}$$

and

$$(\beta_o - k)\, L = 2\pi\,. \tag{15}$$

The antenna thus has one cycle of amplitude modulation, with zero amplitude at each end. The corresponding wave spectrum is shown in Fig. 3c; the calculated field pattern for this case has a side-lobe level less than 5% of the main-lobe amplitude.

An experimental model of a driven end-fire array of 15 wavelengths (sketched in Fig. 2c) was built in order to study the possibility of attaining an amplitude-modulated antenna at essentially constant phase velocity. The philosophy of the design was to employ a TEM transmission line periodically loaded by closely spaced short dipoles. With the dipoles much shorter than resonant length, each would act primarily like a small capacitive admittance across the line. As can be readily shown, the input conductance of a short center-driven dipole is proportional to the 4th power of the length, but the susceptance is only directly proportional to the length. Thus, slight tapering of the lengths of the elements should

(Continued on page 27)

Research and Development of Electronic Mapping

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Research Instructor in Civil Engineering

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A recent research and development contract between the University and the City of Spokane Urban Renewal Department is the basis for an entirely new type of research in both computer applications and urban analysis. This project involves the electronic positioning of selected information from a large universe of data stored on magnetic tape. The actual mapping of the information can be effected either through use of conventional electronic printout equipment, such as the IBM Model 717, or photographed from an image displayed on a cathode-ray tube. In either method, the data shown must be interpreted through an overlay grid or an outline map of some type that enables the viewer to interpret the positioned data in terms of a frame of reference. The superimposition of the reference screen or base map is generally effected in the photographic stage by the use of an overlay map, drawn on clear acetate, showing some features of the city. The base map or grid can also be developed by using an electronic scanner to record the base-map outline and then displaying the base map simultaneously with other desired information on the cathode-ray tube.

The construction of maps through electronic techniques is by no means new. The U.S. Weather Bureau pioneered what is known as a "printout" system for representing certain weather information collected from field stations covering a broad expanse of the earth's surface. However, most of the development of electronic mapping to date has been achieved with machines that actually plot lines. These machines generally make use of an inking head that may be directed to any spot on a plane surface by utilizing either punched cards or taped information. The lines are constructed as vectors, and curved images are possible only by the use of a series of very short straight-line elements. The disadvantage of the ink plotter is that it is extremely slow when compared to either a printout or cathode-ray tube display; nevertheless, it has great utility in many fields. One example of how it is being used in the construction of "traffic trees," a series of desire lines of flow tracing the pattern of movement from any designated traffic-enumeration zone to all other zones of the system. A desire line is a straight line drawn from the point where the traveler begins his trip to his destination. Not only can these straight lines be generated to represent desires-of-flow direction, but a width band of parallel lines may also be made to represent the volume of traffic flow. This type of plotter can also stamp numbers any place on the plane surface, indicating quantities such as traffic volumes or ground elevations.

C. D. Rogers

The cathode-ray tube may also be used to show desire lines of travel by the generation of the vector on the surface of the tube representing the origin and destination of the trip. In the Chicago Area

EXAMINING THE MAPPED OUTPUT FROM THE MACHINES IN THE NORTHWEST RESEARCH COMPUTER LABORATORY

STUDYING A TRANSPARENCY

The base map, outlined on a clear acetate film, is superimposed onto the machine printout to produce a map like that shown in the next photograph.

Transportation Study this technology was used to record the traces of all daily trips made in the Chicago area on a typical day. A total of 2900 quarter-square-mile grid units were used to define the points of origin and destination of the desire trips. Each trip showed as a lighted trace on the cathode-ray tube, and some degree of total trip volume could be comprehended from a photographic recording of the total amount of light displayed by the 400,000 trips simulated on the face of the tube. This particular job, which took 3½ hours of machine time, represented information that could never have been done by hand and that also showed many characteristics of travel never before viewed. For example, when all of the trips between 8:00 and 9:00 p.m. were simulated and displayed photographically, the viewer could get a fairly good idea of the trade area from which the patrons came to each of the local shopping centers. These procedures were developed in 1959.

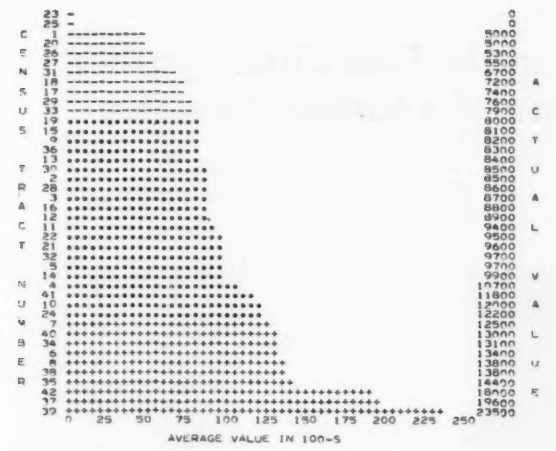
The University of Washington project on electronic mapping is unique for several reasons. First, it deals with a universe of several million pieces of information representing many different types of measurements used in urban analysis. Second, it is developing a technology for mapping much of this information in symbolic form, representing an event either by a symbol on a map or by symbols indicating some scale of intensity for the events recorded. Third, by the use of bar charts and distribution curves, it shows alongside the map statistical information that enables the viewer to see at one glance not only that portion of the distribution curve he is looking at, but also where that portion of the distribution curve or data array is located in space. (These have been termed *statistographs*).

An interesting sidelight of the project is the development of a program that will translate a street address into a position on the map. In this way it is possible to show very quickly any information that is recorded by an address, such as a particular type of industry, a police call, a case of illness representing a particular disease, and many other types of information. For example, one could trace the spatial aspects of an epidemic through periodic runs of cards on which the home addresses of the diseased people were punched. The importance of this address-translation program, however, lies in the fact that it can immediately print out information from cards that have already been collected for other purposes by public agencies and private firms.

THE MAP, SHOWING SOUND DWELLING UNITS AS A PERCENTAGE OF TOTAL DWELLING UNITS

On this early output, the numbers show percentages, computed and positioned automatically; the program also prints out the map legend just given, as well as the north arrow and scale.

Because the client agency for the particular research and development under discussion is an urban-renewal department of government, the mapping contemplated by this project relates mainly to information significant to an urban-renewal program. In fact the mapping development is part of a larger program, called a Community Renewal Program (CRP), that the City of Spokane is currently undertaking. This program, which receives two thirds of its financial support from the Federal government, is designed to study the long-range total urban-renewal needs of a city and to set up criteria for the delimitation and scheduling of specific urban-renewal projects. The data collection for the Spokane CRP,



ARRAY INDICATING MEDIAN LAND VALUE

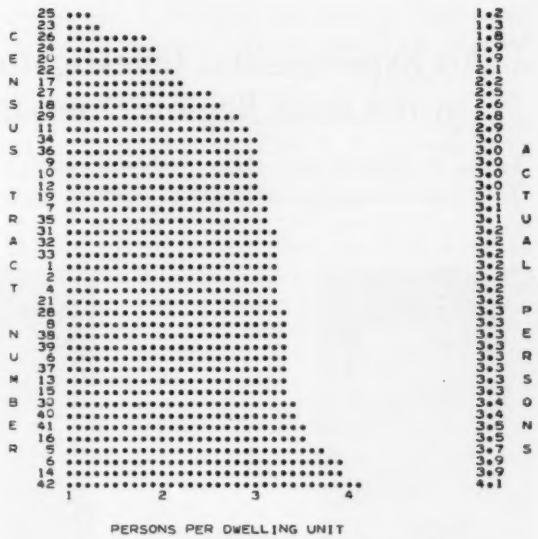
Here the program requires different symbols for the extreme quartiles; subsequently these symbols will show the spatial occurrence of quartiles graphically: viewing graph and map together will combine graphical and related spatial data.

under the general supervision of the University personnel concerned with the electronic mapping project, has involved the development of over 70,000 punched cards representing information on every piece of property in the city. This information includes the type of use of the property; the ground area of the lot; assessed valuation; floor area, age, and condition of the building; and other pieces of information relevant to urban-renewal study. Additional information is fed into the storage system from census data available from the Federal government and data collected by the State of Washington through the Employment Security Department and the Tax Commission. After the punched data are recorded on cards, the information is transferred to magnetic tape from which various summaries and computations can be made in connection with the mapping process.

At the time of this writing the data-collection phase has been concluded and experimental maps have been produced. An estimate indicates that for as little as 15 or 20 cents this method can produce maps that would cost up to $100 to produce by hand.

An interesting aspect of the technology is that hundreds of maps, if not thousands, can be produced as a basis on which to develop a screening for observing conditions that may or may not be significant to examine in greater depth. In contrast, with hand-constructed maps so much money is committed to the production of the map that certain conclusions must be drawn before it is made.

In another sense also the University project is unique. The contract the University has with the City of Spokane came about through a broadened purview of the National Housing Act in terms of



ANOTHER ARRAY FROM 1960 SPOKANE CENSUS

Arnold Rom developed a program that plots in a minute 12 of these bar charts, which determine limits for information to be requested in later mapping; this permits changing the abscissa *scale*, also handling either percentages or absolute values.

developing both research and analytical techniques that will aid in the long-term evaluation and planning of housing development as well as other urban amenities like open space and mass rapid transit. This is the first time a university has been involved in a city research and development program financed principally out of Federal funds administered by the Housing and Home Finance Agency. As such it represents a new avenue of development in urban research and analysis that will be watched closely throughout the country. The contract calls not only for development of the mapping technology at the University, but also for the analysis of the data itself in whatever ways machine technology can assist. In the latter respect, the aim is to define new models of urban growth and development that will have general utility for all cities of the country.

The authors are the general director and supervisor of the project. Mr. Arnold Rom, of the Boeing Company, is the programming consultant. The facilities of the Northwest Computing Laboratory at the University of Washington are being used in the project. The mapping-development and data-collection phases of the Spokane CRP amount to approximately $30,000. As a consequence of the interest many agencies have expressed in the technology since the beginning of the project, its principal personnel will be conducting a workshop on electronic mapping at the national convention of the American Institute of Planners in Detroit during November, 1961.

(Bibliography on page 27)

An Experimental Investigation of the Flow Characteristics in the Base Region Downstream of a Surface Back Step

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When a supersonic stream flows past a body with a blunt base or past a surface back step, fluid in the base region downstream of the step is entrained by the external stream. This tends to reduce the pressure at the base and cause the external flow to turn toward the base region. If the mass that is entrained cannot be replaced, a closed wake is produced in which the trapped fluid merely recirculates. Figures 1a and 1b show two flow patterns corresponding to this situation. If mass is injected into the base region or removed from it, the general nature of the flow remains the same. However, the pressure will be altered to an extent that depends upon the amount of fluid injection or removal. The exact flow pattern in the base region will depend upon the method of injection or removal.

A knowledge of the pressures, temperatures, and flow velocities found in such regions and the ability to predict them are important to organizations engaged in aero-space activities. Consequently, problems of this general nature have been the subject of extensive investigation during the past few years.

This article reports preliminary results of an experimental investigation of the static pressure, total pressure, and velocity distributions in one such region, that is, a closed wake in an internal two-dimensional flow channel like that shown in Fig. 1a.

As far as gross effects that take place in back step or base region are concerned, Korst and associates at the University of Illinois have already developed a highly satisfactory means of theoretically predicting base pressures and base temperatures for a wide variety of body geometries, with and without mass bleed. For the problem of determining the base pressure in a closed wake, the University of Illinois group has based its analysis on a flow model of the type shown in Fig. 1a. (See Ref. 1 for models used in the treatment of more complex flow cases.)

The flow model shown in Fig. 1a consists of the following components:

1. Supersonic flow at cross section 1.
2. An expanding flow between cross sections 1 and 2.
3. A mixing region between the adjacent external stream and the base region.
4. A recompression region between 3 and 4.

In using the model the following assumptions are made:

1. The pressure in the base region and the mixing region is at a constant value imposed by the pressure in the adjacent external free stream.
2. The flow undergoes a Prandtl-Meyer expansion between 1 and 2, the amount of turn depending upon the Mach number and pressure at 1 and the pressure in the base region.
3. The flow direction of the adjacent external stream is determined by the potential flow solution for expansion from the conditions at cross section 1 to the pressure in the base and mixing regions.

* Mr. Strout and Mr. Nicholson have been with the Boeing Company since graduation. This article is based upon a joint thesis written for their MS degrees in Mechanical Engineering, which they received in June, 1961.

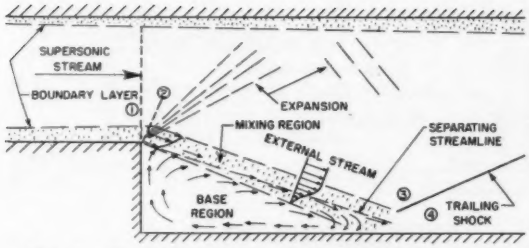

FIG. 1a. FLOW PATTERN AT A SUDDEN EXPANSION IN AN INTERNAL SUPERSONIC FLOW CHANNEL

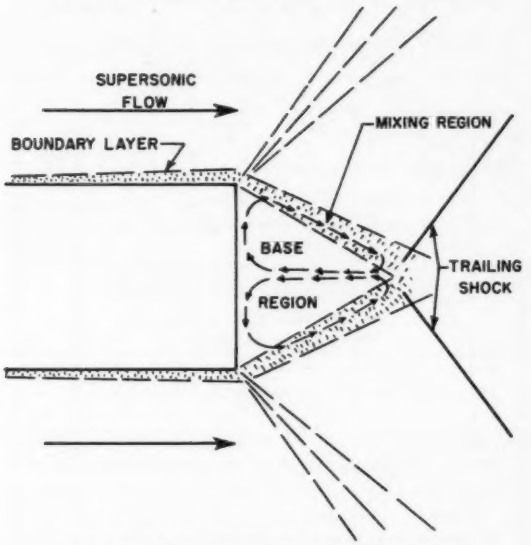

FIG. 1b. SUPERSONIC FLOW PAST A BODY WITH A BLUNT BASE

4. The velocity profiles in the mixing region can be predicted by the compressible jet-mixing theory given in Ref. 4.
5. The pressure rise between regions 3 and 4 may be determined by consideration of flow of the adjacent stream through an oblique trailing shock, the strength of which depends upon the Mach number and flow direction of the adjacent external stream and on the downstream wall angle.
6. Only that flow in the mixing region for which the stagnation pressure is equal to or greater than the static pressure in region 4 can pass downstream. Flow along streamlines at lower total pressure will merely turn back and recirculate in the closed wake.

Details of the theoretical considerations involved in obtaining a solution to the base-pressure problem are too lengthy to include here. Essentially, however, a solution for the base pressure for a flow such as that shown in Fig. 1a may be obtained for given flow conditions at 1 in the following manner: First a base pressure is assumed. This establishes a flow direction and Mach number of the adjacent stream as well as the pressure rise between regions 3 and 4. The velocity profile in the mixing region is then determined by the mixing theory given in Ref. 4. Then the resulting theoretical flow pattern is examined to determine whether the condition of conservation of mass in the wake is satisfied. That is, with the direction of the adjacent flow known, the continuity equation can be used to determine the "separating" streamline in the mixing region that separates the main flow from the recirculating flow. The total pressure for this streamline is then determined. If it is equal to the static pressure calculated for region 4 downstream of the trailing shock, the solution is determined. If the pressure condition is not satisfied, a new base pressure is assumed and the process repeated until the pressure requirement is met. The streamline for which the pressure condition is met is called the "discriminating" streamline. For cases of mass bleed into or out of the base region, the general approach is the same except that allowance must be made for the mass bleed or removal in using the continuity equation. Here, of course, the separating streamline and the discriminating streamline are different.

The high degree of success attained with the theory described above indicates that the flow model used in the analysis does represent the actual flow quite well. However, further information on the details of the flow pattern and on the influence that deviations from assumed conditions of the various components of the flow model have upon the solution of the problem is important. In addition, the lack of compressible mixing data makes it impossible to take into account the influence of incompletely developed mixing-region profiles on the theoretical base-pressure solution. Further study of the details of the flow may provide useful mixing data.

The experimental data obtained in this investigation represent preliminary results of what is planned as a more extensive investigation of flow characteristics in base regions. Although this investigation was limited to a closed wake with a single step height, attempts will be made in future work to determine Mach number and total temperature profiles, for both closed wake flows and flows with mass injection or removal.

APPARATUS

A schematic drawing of the two-dimensional sudden-expansion test apparatus used in the investigation is shown in Fig. 2. A half nozzle was used so that the significant dimensions could be as large as possible. The nozzle was designed for $M = 1.775$ just ahead of the sudden expansion, with no allowance

being made for boundary-layer development in the nozzle. The nozzle throat height was 0.225 in.; the channel width, 0.5 in.; and the height of the back step, 0.4 in. Both the nozzle blocks and the two side plates for the apparatus were constructed of lucite so that visual observations could be made.

One of the side plates was attached permanently to the nozzle-block sections. The other plate was constructed so that it could be moved along the side surface of the nozzle blocks in a direction normal to the axis of the nozzle. This allowed very detailed readings of static pressures at the sidewall to be made with only a few static pressure taps. More important, it allowed total pressure probes inserted into the flow channel through the sidewall to be moved in very small increments normal to the nozzle axis. This permitted detailed traverses across the base and mixing regions.

Two total pressure probes were used. One consisted of a 1/16-in. metal tube inserted with a squeeze fit through the movable sidewall and extended to the inside surface of the opposite wall. The end of the tube in contact with the opposite wall was sealed, and a number 80 hole was drilled ¼ in. from the end so that the hole was half way across the channel. This probe was used for measurements in the base region.

The second probe was constructed from a 0.02-in. stainless-steel tube soldered to form an extension of a 1/16-in. tube. The 1/16-in. tube was inserted through the movable sidewall with a squeeze fit in such fashion that the end of the tube was flush with the inside surface of the wall. The 0.02-in. extension tube was bent into a 90-deg elbow so that the first ¼ in. was normal to the sidewall, and the remaining portion of the tube (also ¼ in.) was parallel to the sidewall and half way across the channel. By turning the 1/16-in. tube the hole in the total-pressure probe could be made to face in any direction desired parallel to the sidewall. This probe was used in the mixing region, the base region, and the nozzle exit plane.

All holes for static pressure taps were drilled with number 80 drills. Pressures were recorded on U-tube mercury manometers.

TEST PROCEDURE

Static Pressures

Static pressures were measured in the nozzle exit plane; in the base, mixing, and recompression regions; and in the flow outside the mixing region. The static-pressure taps were positioned by moving the sliding sidewall in 1/16-in. increments.

Total Pressures

Total pressures were measured in the nozzle exit plane, and in both the mixing and base regions.

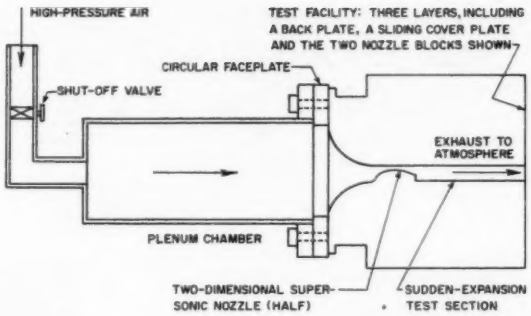
HIGH-PRESSURE AIR
TEST FACILITY: THREE LAYERS, INCLUDING A BACK PLATE, A SLIDING COVER PLATE AND THE TWO NOZZLE BLOCKS SHOWN
CIRCULAR FACEPLATE
SHUT-OFF VALVE
EXHAUST TO ATMOSPHERE
PLENUM CHAMBER
TWO-DIMENSIONAL SUPERSONIC NOZZLE (HALF)
SUDDEN-EXPANSION TEST SECTION

FIG. 2. SCHEMATIC DRAWING OF APPARATUS

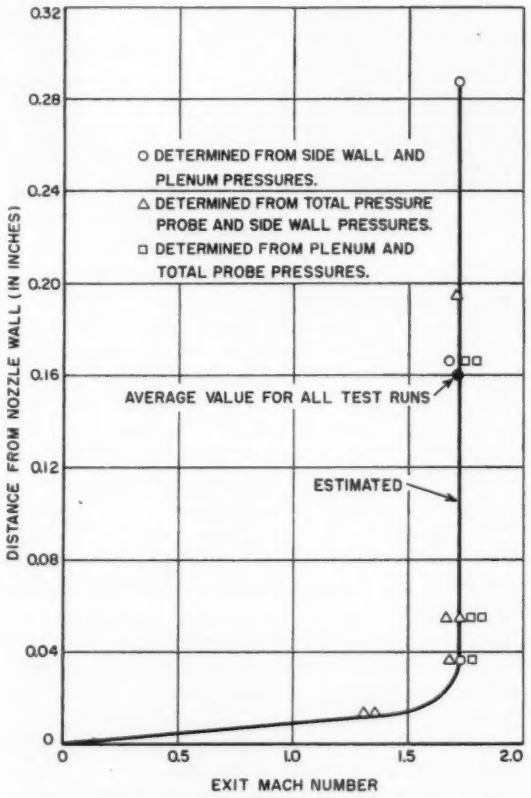

FIG. 3. MACH-NUMBER PROFILE AT NOZZLE EXIT

1. ***Nozzle Exit Plane.*** The 0.02-in. total-pressure probe was placed just inside the exit plane of the nozzle, the tube axis being aligned with the nozzle axis. A traverse was then obtained by moving the side plate. Static pressures in the exit plane and the plenum were measured at the same time.

2. ***Mixing Region.*** The mixing region was traversed with the 0.02-in. total-pressure probe at two stations about half way downstream in the mixing

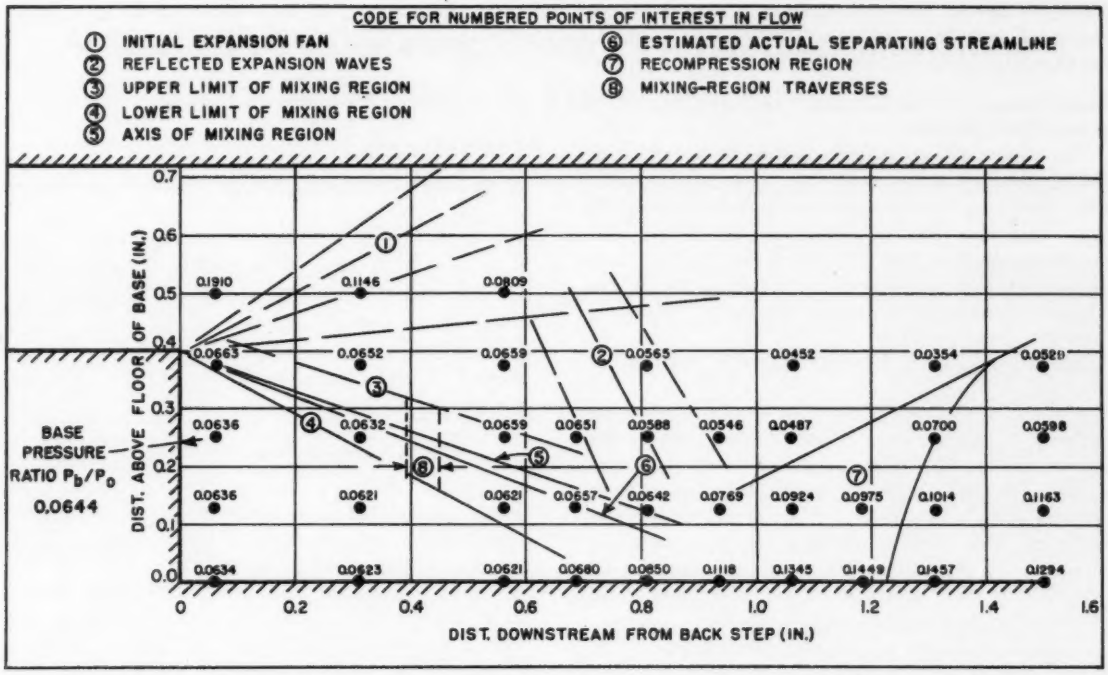
CODE FOR NUMBERED POINTS OF INTEREST IN FLOW
① INITIAL EXPANSION FAN
② REFLECTED EXPANSION WAVES
③ UPPER LIMIT OF MIXING REGION
④ LOWER LIMIT OF MIXING REGION
⑤ AXIS OF MIXING REGION
⑥ ESTIMATED ACTUAL SEPARATING STREAMLINE
⑦ RECOMPRESSION REGION
⑧ MIXING-REGION TRAVERSES
DIST. ABOVE FLOOR OF BASE (IN.)
BASE PRESSURE RATIO P_b/P_0 0.0644
DIST. DOWNSTREAM FROM BACK STEP (IN.)

FIG. 4. STATIC PRESSURE RATIOS AT SIDEWALL OF TEST SECTION (PLENUM PRESSURE USED AS REFERENCE)

region. Traverses were made for several deflection angles of the probe for each station so that flow direction in the mixing region could be determined and the probe aligned with that direction. Although the traverses were not made in a direction normal to the flow in the mixing region, by using interpolation between the data for the two stations, estimates could be made of a total pressure profile normal to the mixing region. As the total pressure readings were being taken, static pressures at several sidewall positions and the pressure in the plenum were also recorded.

3. ***Base Region.*** The total pressure was measured at several positions in the base region by means of the 1/16-in. and the 0.02-in. probes. The probes were turned through a range of deflection angles to determine as well as possible the flow direction. Concurrent readings were taken of the static pressures at the sidewall and of the plenum pressure.

Visual Observation

Two sets of visual observations were made of the flow. In the first, threads were attached to probes inserted into the channel to help confirm indicated flow directions observed previously. For the second set of observations, water containing a high concentration of soap was introduced through the sidewall into both the base and mixing regions. This produced soapy traces on the solid surface, indicating the flow patterns near these surfaces.

RESULTS

Some of the data obtained in this investigation duplicate data that previous investigators have recorded. For the sake of completeness this information is repeated here and discussed briefly. No attempt is made in this article to discuss the effect on the base pressure solution of the observed deviations of the actual flow from the theoretical flow model. A report on these effects will be made when more complete data are available.

The more significant results of the investigation are presented in graphical form in Figs. 3, 4, and 5. These results are discussed in the sections that follow.

Flow at the Exit Plane of the Nozzle

The approximate Mach-number profile at the exit plane of the nozzle is shown in Fig. 3. Determinations of Mach number were based on measurements of static pressure at the sidewall and plenum pressure, on measurements of sidewall static pressure, and of total pressures obtained with the 0.02-in total-pressure probe, and on measurements of the plenum pressure and total pressure in the stream. Apparently the use of the half-nozzle did not appreciably affect the uniformity of the profile. The thickness of the

boundary layer at the nozzle wall ahead of the sudden expansion was about 0.04 in. The velocity profile in the boundary layer was not determined precisely.

Static Pressures at the Sidewall of the Flow Channel

The static pressures obtained at the sidewall of the test section are shown in Fig. 4. The overall results agree well with less detailed observations made in the past. The base pressure agrees almost exactly with results reported in Ref. 1. Also shown in Fig. 4 are the approximate boundaries of the mixing region, the recompression region, and the expansion emanating from the corner.

The data on static pressures may be summarized as follows:

1. Static pressures in the base region were quite uniform. Along the floor of the base region near the end of the mixing region, pressures show an increase, indicating the effect of the recompression of the flow as it is turned back toward the base region. Similarly, at the face of the back step and extending outward into the base region a short distance, a recompression effect produced by the interaction of the recirculating flow with the step is shown.

2. Static pressures at the high-velocity side of the mixing region were higher than those at the low-velocity side. The variation across the mixing region was about 7 to 10% of the average pressure in the region. In terms of the actual pressure the variation was only about 0.5 psia. Pressures decreased down the length of the mixing region, the variation being less than 0.5 psia, or less than about 5% of the average pressure in the region. Some inconsistency in static pressure readings along the length of the mixing region was observed, probably owing to the presence of weak waves reflected from the top of the flow channel.

3. The static pressures observed in the vicinity of the trailing shock indicate the complexity of the flow in the region. The recompression process apparently extended over a considerable length, and for the particular step height used in this investigation was influenced considerably by the expansion waves reflected from the top of the channel. The flow in the recompression zone appears to show the greatest deviation from the idealized flow pattern assumed for the simplified model used in the base-pressure theory.

Flow in the Mixing Region

As was pointed out in the discussion of test procedure, traverses of the mixing region were made in a direction normal to the nozzle axis. The positions at which these traverses were made are shown in Fig. 4. The Mach-number profile normal to the flow direction in the mixing region was then obtained by interpolation of the data from the traverses. Figure 5

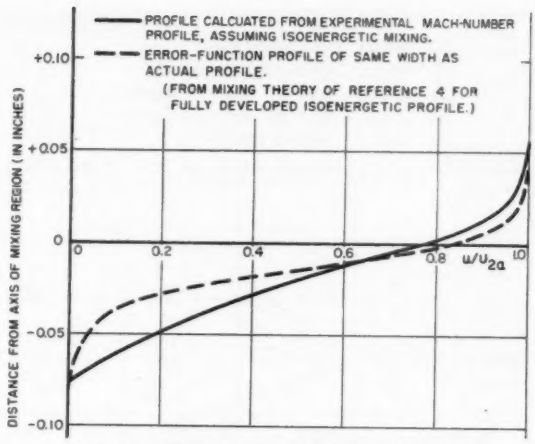

FIG. 5. DIMENSIONLESS VELOCITY PROFILE IN MIXING REGION

shows a calculated form of this profile expressed as a dimentionless velocity, with U_{2a}, the velocity of the adjacent stream, used as the reference velocity. The profile was calculated from the Mach-number profile, under the assumption that the shear layer is at constant stagnation temperature. In future work attempts will be made to measure the distribution of the stagnation temperature in the mixing zone.

Also shown in Fig. 5 is an error-function profile whose width is the same as that of the observed profile. The error-function distribution is the theoretical mixing solution obtained by the methods of Ref. 4 for fully developed profiles. Comparison of these two profiles shows that the actual profile was not fully developed. This result would be expected, considering the position at which the profile was obtained.

Figure 4 shows the approximate boundaries of the mixing region. The location of the boundaries was estimated from information on the profile in the nozzle exit plane and the one available interpolated mixing profile. The approximate location of the physical separating streamline is also shown in Fig. 4. This was calculated by satisfying continuity requirements between the cross section just downstream of the corner and the cross section at which the traverse was made. Since conditions downstream of the corner were not measured, approximate values were determined by assuming an isentropic expansion of the flow from the observed conditions in the nozzle exit plane. Although the estimates of the location of the separating streamline are in no sense precise at this point, the results obtained do offer further support of the correctness of the mixing component of the base model.

An encouraging aspect of this investigation was that a probe could be inserted into the mixing region

(Continued on page 27)

A Method of Estimating Creep Deformations and Stress Relaxations in Plane Structures*

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To meet increasing demands that the region of design allowables of structures be extended, various studies of creep deformation and stress relaxation caused by such deformation have been published. (See for examples Ref 2 and 3.) This paper develops a method for estimating the creep deformation and stress relaxation in plane structures subjected to arbitrary in-plane loads and complicated boundaries. The basis for this development is the direct-stiffness method described by M. J. Turner *et al* for the elastic analysis of complex structures (Ref 4 and 5). The advantage of following this procedure is that many of the computer techniques developed for the elastic analysis of structures can then be adapted to the analysis of creep deformation.

Elastic Analysis

For this paper the reader should keep the following qualifications in mind:

1. The temperature in the plane stucture is raised and the loads are applied to the structure in steps. The temperature distribution is not necessarily uniform throughout the structure and may change gradually with time.

2. The plastic strains incurred during the period of loading and the tertiary creep strain are not considered in this analysis. The change in geometry of the plane structure is thus assumed to be sufficiently small that the second and higher-order terms involving products of creep strains and creep strain rates can be considered negligible.

3. The structural material when subjected to primary creep follows the biaxial stress-strain rate law proposed by Hoff (Ref 6). When the material is subjected to secondary creep, it follows the stress-strain rate law proposed by Odquist (Ref 7).

4. Structural instability due to creep deformation is not considered.

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The basic element considered in this paper is a triangle having a uniform distribution of stresses. Figure 1 shows this triangular element with nodes 1, 2, and 3.

For a structure that is initially stress free, the elastic deformations and stresses upon loading and heating can be determined by the procedure outlined in Ref 5. The triangular elements of the plane structure are first assumed to be disconnected and then are subjected to thermal expansion due to the temperature rise in the structure. This need not be uniform as long as the temperature can be considered uniform within each triangular element. The elements are then forced back to their original shapes, reconnected, and loaded by the externally applied surface tractions while satisfying the displacement boundary conditions where such conditions are prescribed. The resultant nodal forces, $\{F\}$ on the triangular element, are expressed in terms of the elastic displacements $\{u^e\}$ owing to external loads as

$$\{F\} = \{F^o\} + [K]\{u^e\}, \qquad (1)$$

where the nodal forces, $\{F^o\}$, are required to compress the initially expanded triangular elements to their original shape. (See Ref 5.)

By assembling Eq (1) for every triangular element in the plane structure, a relation between the column matrix of the resultant nodal forces and the column matrix of the elastic nodal displacements is obtained as follows:

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* This paper is a result of investigations being made at the Aero-Space Division of the Boeing Company, Seattle, Washington. An identical topic with a similar approach is treated in a report by Padlock *et al* (Ref 1).

** Mr. Weikel was formerly Associate Professor of Aeronautical Engineering.

$$\{F_R\} = \{F^o_R\} + [K_R]\{u^e_R\}. \qquad (2)$$

The resultant initial nodal forces, $\{F^o_R\}$, can be computed by determining the initial nodal forces, $\{F^o\}$, of the individual triangular elements, by adding all nodal forces at each node. In Eq (2), $[K_R]$ is now the assembled stiffness matrix of the plane structure.

Among the terms in Eq (2), the elements of the assembled stiffness matrix, $[K_R]$, are known; but the resultant elastic displacements, $\{u^e_R\}$, remain unknown except where the displacements are prescribed on the boundaries. As for $\{F_R\}$, the resultant nodal forces of the interior nodes that are in internal equilibrium naturally vanish. In addition, $\{F_R\}$ is known on the boundaries where forces are prescribed. Therefore Eq (2) will determine $\{u^e_R\}$. Since the elastic displacements, $\{u^e\}$, at each node are known, the corresponding nodal forces acting on the three nodes of each triangular element can be determined by use of Eq (1).

The uniform state of stress in the triangular element in Fig. 1 owing to nodal forces F_{x1}, F_{x2}, F_{x3}, F_{y1}, F_{y2}, and F_{y3}, can then be determined by use of the following equations of the nodal forces vs stresses:

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \\ F_{x3} \\ F_{y1} \\ F_{y2} \\ F_{y3} \end{Bmatrix} = \begin{bmatrix} y_{23} & 0 & -x_{23} \\ y_{31} & 0 & -x_{31} \\ y_{12} & 0 & -x_{12} \\ 0 & -x_{23} & y_{23} \\ 0 & -x_{31} & y_{31} \\ 0 & -x_{12} & y_{12} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \qquad (3a)$$

or

$$\sigma_{xx} = \frac{x_{31}F_{x1} - x_{23}F_{x2}}{Ah},$$

$$\sigma_{yy} = \frac{y_{31}F_{y1} - y_{23}F_{y2}}{Ah}, \qquad (3b)$$

and

$$\sigma_{xy} = \frac{y_{31}F_{x1} - y_{23}F_{x2}}{Ah},$$

where $x_{23} = x_2 - x_3$, $x_{31} = x_3 - x_1$, $x_{12} = x_1 - x_2$, $\qquad (4)$

$y_{23} = y_2 - y_3$, $y_{31} = y_3 - y_1$, $y_{12} = y_1 - y_2$,

and

$$A = \tfrac{1}{2}(x_{31}\,y_{23} - x_{23}\,y_{13}). \qquad (5)$$

This procedure is a very brief review of the direct-stiffness technique used to determine the elastic deformations and the elastic stresses in each triangular element resulting from external loading and heating of a plane structure.

Creep Analysis in the Plane Structure

After the results of the elastic analysis of the plane structure at the instant of loading are determined, the next step is to determine the amounts of creep deformations and stress relaxation that will occur during a subsequent interval of time.

Creep Displacement Rates. The approach used in this paper is to assume that the creep strain rates are related to the stress deviators. Knowing the stress distributions throughout the plane structure from previous analysis, we can now determine the creep strain rates by using the above biaxial strain-rate law. Since the creep strain rates, $\{\dot{e}'\}$, are linearly related to the creep displacement rates, $\{\dot{u}'\}$, these creep displacement rates are linearly related to the forces; and we may write

$$\{F\} = [H]\{\dot{u}'\}, \qquad (6)$$

where $[H]$ will be called the creep stiffness matrix for the triangular structure.

Since $\{F\}$ is known from the previous analysis, apparently $\{\dot{u}'\}$ can be determined by the use of Eq (6). However, if this equation is written to include the constraining nodes, the creep stiffness matrix $[H]$ is singular and therefore cannot be inverted to solve for the creep displacement rates, $\{\dot{u}'\}$. If these rates are zero at the nodal constraints of a triangular element, and if Eq (6) is written for only the nodal degrees of freedom, the creep displacement rates, $\{\dot{u}'\}$, can now be determined from Eq (6) by simply eliminating the respective rows and columns of the creep stiffness matrix, $[H]$, corresponding to the nodal constraints. With the assumption that the total strain is equal to the sum of the elastic and creep strains, $e = e^e + e'$, and then by recalling the basic assumption that the elastic and creep strains are uniform throughout the triangular element, we can prove that the total displacement is equal to the sum of the elastic and creep displacements, or $u = u^e + u'$. Now, if we further assume that the elastic displacements at a common node are identical for all elements sharing this node, then it follows that the creep displacements are identical at a common node. This assumption appears reasonable when one

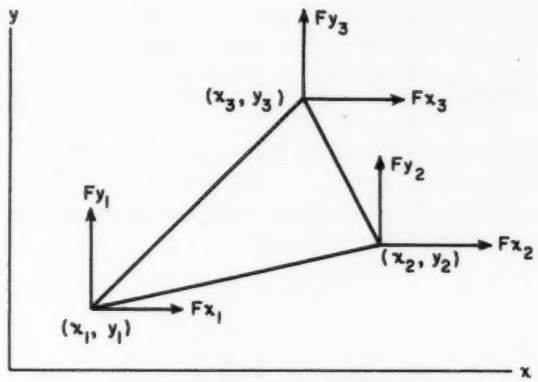

FIG. 1. TRIANGULAR PLATE ELEMENT

considers that it is certainly true when $u'=0$ and when $u^e=\text{const}$.

Therefore we can follow the procedure in elastic-stiffness technique by combining all creep stiffness matrices of Eq (6) for the entire plane structure and then solve for the creep displacement rates as

$$\{\dot{u}'_R\}=[H_R]^{-1}\{F_R\}. \tag{7}$$

The inversion of the above combined creep stiffness matrix, $[H_R]$, is possible when the minimum number of constraints are applied to the creep displacement rates, $\{\dot{u}'_R\}$, so as to prevent rigid body displacements and rotation in the structure. For the plane structure considered here, three constraints in the creep displacement rates will be sufficient to make this combined creep stiffness matrix non-singular. For any additional constraints on the structure, the total displacement rates are set equal to zero, $\dot{u}=\dot{u}'+\dot{u}^e=0$, and as a result the creep displacements rates are equal to the negative of the elastic displacement rates, or $\dot{u}'=-\dot{u}^e$, at such additional constraints.

Elastic Displacement Rates. The second step is to determine the elastic displacement rates. Returning to Eq (1) and differentiating this equation with respect to time yields

$$\{\dot{F}\}=\{\dot{F}^o\}+[\dot{K}]\{u^e\}+[K]\{\dot{u}^e\}. \tag{8}$$

In the above equation, $[\dot{K}]\{u^e\}$ involves the cross-products terms of the displacements and displacements rates; therefore this term is assumed to be negligible with respect to the other terms. Combining Eq (8) for all nodes throughout the plane structure yields

$$\{\dot{F}_R\}=\{\dot{F}'_I\}+[K_h]\{\dot{u}^e_R\}. \tag{9}$$

At the nodes having freedom of displacement, the forces $\{F_R\}$ are prescribed and the displacements $\{u^e_R\}$ are unknowns; at the constraints, the displacements are prescribed and the forces $\{F_R\}$ are unknowns. Similar statements may be made about the force rates $\{\dot{F}_R\}$ and the displacement rates $\{\dot{u}^e_R\}$. Therefore the matrix equation may be partioned to read as follows:

$$\begin{Bmatrix}\dot{F}_{RI}\\ ---\\ \dot{F}_{RII}\end{Bmatrix}=\begin{Bmatrix}\dot{F}^o_{RI}\\ ---\\ \dot{F}^o_{RII}\end{Bmatrix}+\left[\begin{array}{c:c}K_{11} & K_{12}\\ \hdashline K_{21} & K_{22}\end{array}\right]\begin{Bmatrix}\dot{u}^e_{RI}\\ ---\\ \dot{u}^e_{RII}\end{Bmatrix} \tag{10}$$

where $\{\dot{F}_{RI}\}$ and $\{u^e_{RI}\}$ are given

and

$\{\dot{F}_{RII}\}$ and $\{\dot{u}^e_{RI}\}$ are unknowns.

Equation (10) constitutes a system of simultaneous equations that may be solved for the unknowns

$\{\dot{u}^e_{RII}\}$ and $\{\dot{F}_{RII}\}$, provided $K_{11}\neq 0$.

Resultant Displacements. With $\{\dot{u}^e_R\}$ and $\{\dot{u}'_R\}$ known, we can compute the resultant total displacements for a given time interval, Δt, with the help of Eq (9) as

$$\{u_R\}_{t=\Delta t}=\{u_R\}_{t=0}+\Delta t\{\dot{u}_R\}_{t=0}. \tag{11}$$

The total displacement rate, $\{\dot{u}_R\}$, in the above equation is evaluated for constant stress of $\sigma_{t=0}$ during the time interval of Δt. Obviously, the smaller the time interval, Δt, the better the accuracy of $\{u\}_{t=\Delta t}$ determined by this procedure. This use of incremental creep displacements, $\{\dot{u}_R\}_{t=0}\Delta t$, for a given time interval makes the method of approach used in this paper adaptable for estimating creep deformations in plane structures subjected to variable load and temperature history.

Stress Relaxations. By using Eq (8) and the results of the elastic displacement rates from Eq (9), the force rate for each triangular element can be determined. And finally, for a given time interval of Δt, the nodal force after time Δt is

$$\{F\}_{t=\Delta t}=\{F\}_{t=0}+\Delta t\{\dot{F}\}_{t=0}. \tag{12}$$

With the nodal forces $\{F\}$ known, we can now compute the stresses in the triangular element by Eq (3). Again, the force rate, $\{\dot{F}\}$, in the above equation is evaluated for a constant stress of $\sigma_{t=0}$ during the time interval of Δt.

It should also be noted that the procedure indicated by Eq (10) and (12) is based on the assumption that the creep strain rate at any time can be expressed by the time-hardening relation. This relation assumes that under varying stresses the creep strain is a function of the current stresses and time, whereas another popular concern, the strain-hardening relation, assumes that the creep strain rate is a function of the current stresses and creep strain. In this development, the time-hardening theory was favored over the strain-hardening theory because of its mathematical simplicity. Figure 2 shows schematically the creep strains determined by this procedure of incremental time-hardening theory.

Creep Stiffness Matrix

Consider a triangular element with arbitrary node location as shown in Fig. 1. We assume that the strain rates are approximately constant throughout the triangular element and that the creep strain rates can be approximated by

$$\dot{\epsilon}'_{xx}=a,\ \dot{\epsilon}'_{yy}=g,\ \text{and}\ \dot{\epsilon}'_{xy}=b+\mathrm{d}\ , \tag{13}$$

where a, b, d, and g, are the constants associated with the constant strain rates. Then the creep displacement rates, $\dot{u}'$ and $\dot{v}'$, can be expressed as

$$\dot{u}'=ax+by+c$$

and

$$\dot{v}=dx+gy+f, \tag{14}$$

where c and f are additional constants associated with the rigid body-displacement rates. Substituting the coordinates of the three nodes into Eq (14), we can solve for the six constants in terms of the displacements and the coordinates at these three nodes. Substituting these established constants into Eq (13) yields

$$\begin{Bmatrix} \dot{\epsilon}'_{xx} \\ \dot{\epsilon}'_{yy} \\ \dot{\epsilon}'_{xy} \end{Bmatrix} = \frac{[\mathrm{D}]}{2A} \begin{Bmatrix} \dot{u}'_1 \\ \dot{u}'_2 \\ \dot{u}'_3 \\ \dot{v}'_1 \\ \dot{v}'_2 \\ \dot{v}'_3 \end{Bmatrix} \tag{15}$$

where $[D]=\begin{bmatrix} y_{23} & y_{31} & y_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & -x_{23} & -x_{31} & -x_{12} \\ -x_{23} & -x_{31} & -x_{12} & y_{23} & y_{31} & y_{12} \end{bmatrix}$ (16)

Similarly the strain rates vs displacement rates for elastic and total strains are

$$\begin{Bmatrix} \dot{\epsilon}^e_{xx} \\ \dot{\epsilon}^e_{yy} \\ \dot{\epsilon}^e_{xy} \end{Bmatrix} = \frac{[\mathrm{D}]}{2A} \begin{Bmatrix} \dot{u}^e_1 \\ \dot{u}^e_2 \\ \dot{u}^e_3 \\ \dot{v}^e_1 \\ \dot{v}^e_2 \\ \dot{v}^e_3 \end{Bmatrix} \tag{17}$$

and

$$\begin{Bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{xy} \end{Bmatrix} = \frac{[\mathrm{D}]}{2A} \begin{Bmatrix} \dot{u}^e_1 + \dot{u}'_1 \\ \dot{u}^e_2 + \dot{u}'_2 \\ \dot{u}^e_3 + \dot{u}'_3 \\ \dot{v}^e_1 + \dot{v}'_1 \\ \dot{v}^e_2 + \dot{v}'_2 \\ \dot{v}^e_3 + \dot{v}'_3 \end{Bmatrix}. \tag{18}$$

It is now necessary to describe a stress-strain rate relation for a material exhibiting creep behavior. The writers selected the biaxial strain law by Odquist (see Ref 7) for secondary creep because of its mathematical simplicity. For the case of plane stress, the law states that

$$\begin{aligned} \dot{\epsilon}'_{xx} &= cJ'^{m}_2 S_{xx} = cJ'^{m}_2(\tfrac{2}{3}\sigma_{xx} - \tfrac{1}{3}\sigma_{yy})\ , \\ \dot{\epsilon}'_{yy} &= cJ'^{m}_2 S_{yy} = cJ'^{m}_2(\tfrac{2}{3}\sigma_{yy} - \tfrac{1}{3}\sigma_{yy})\ , \\ \dot{\epsilon}'_{xy} &= cJ'^{m}_2 S_{xy} = cJ'^{m}_2 \sigma_{xy} \end{aligned} \tag{19}$$

where $J'_2=(\sigma^2_{xx}+\sigma^2_{yy}-\sigma_{xx}\,\sigma_{yy}+3\sigma^2_{xy})$ is a known quantity computed from known stresses σ_{xx}, σ_{yy}, and σ_{xy}. Material constants c and m are governed by the creep response of the structural material at temperature T.

For a material subjected to primary creep, Hoff showed (Ref 6) that the rate for the stress-strain relation can be expressed as

$$\dot{e}'_{xx}=\frac{c}{q}t^{1/q-1}J'^{m}_{2}S_{xx}\,,$$

$$\dot{e}'_{yy}=\frac{c}{q}t^{1/q-1}J'_{2}S_{yy}\,, \qquad (20)$$

and

$$\dot{e}'_{xy}=\frac{c}{q}t^{1/q-1}J'_{2}S_{xy}$$

where q is an additional constant for a material subjected to creep. Equation (20) involves a time-dependent constant but is otherwise identical in form with Eq (19). Therefore, although subsequent deviations will be made in terms of secondary creep, the results derived from Eq (19) can be extended to primary creep by merely replacing the time-independent constant, c, with the time-dependent constant,

$$\frac{c}{q}t^{1/q-1}.$$

Solving for the stresses in Eq (19) and substituting the results into Eq (3) and (15), Eq (6) in the final expanded form becomes Eq (21):

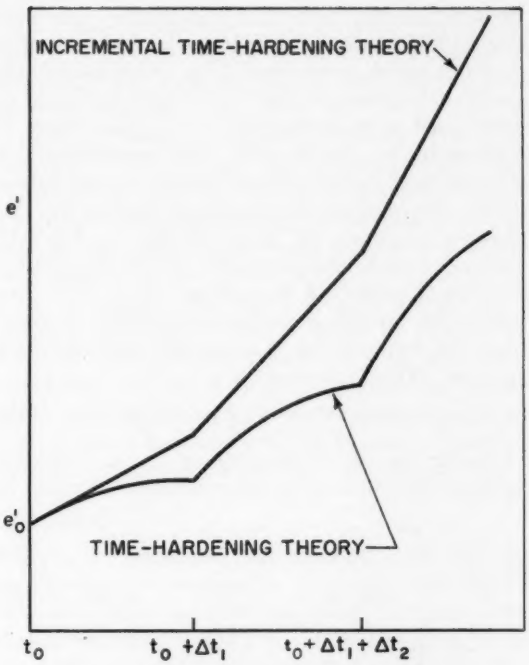

FIG. 2. CREEP STRAIN BY INCREMENTAL TIME-HARDENING THEORY

From Eq (21) we can see that the creep stiffness matrix is a symmetric matrix. The similarity between this creep stiffness matrix and the stiffness matrix shown by Eq (B-3) of Ref 5 should be noted.

Column under Uniaxial Compression

The numerical techniques developed in the previous sections are used to determine the amount of stress relaxation in a column subjected to constant compressive deformation. For the purpose of com-

$$\begin{Bmatrix} F_{x1} \\ F_{x2} \\ F_{x3} \\ F_{y1} \\ F_{y2} \\ F_{y3} \end{Bmatrix} = \frac{h}{4Ac\,J'^{m}_{2}} \begin{bmatrix} x^2_{23}+2y^2_{23} & & & & & \\ x_{23}x_{31}+2y_{23}y_{31} & x^2_{31}+2y^2_{31} & & \text{(Sym)} & & \\ x_{12}x_{23}+2y_{12}y_{23} & x_{31}x_{12}+2y_{31}y_{12} & x^2_{12}+2y^2_{12} & & & \\ -2x_{23}y_{23} & -x_{23}y_{31}-x_{31}y_{23} & -x_{23}y_{12}-x_{12}y_{23} & 2x^2_{23}+y^2_{23} & & \\ -x_{31}y_{23}-x_{23}y_{31} & -2x_{31}y_{31} & -x_{31}y_{12}-x_{12}y_{31} & 2x_{31}x_{23}+y_{31}y_{23} & 2x^2_{31}+y^2_{31} & \\ -x_{12}y_{23}-x_{23}y_{12} & -x_{12}y_{31}-x_{31}y_{12} & -2x_{12}y_{12} & 2x_{12}x_{23}+y_{12}y_{23} & 2x_{12}x_{31}+y_{12}y_{31} & 2x^2_{12}+y^2_{12} \end{bmatrix} \begin{Bmatrix} \dot{u}'_1 \\ \dot{u}'_2 \\ \dot{u}'_3 \\ \dot{v}'_1 \\ \dot{v}'_2 \\ \dot{v}'_3 \end{Bmatrix}$$

parison the relaxation test on Cr-Mo steel by Johnson (Ref 9) was selected as an example for demonstrating this numerical procedure. The compression test specimen as shown in Fig. 3 (a) is initially heated to 525° C and then compressed to a total strain of 0.001 in. per in. See Fig. 3 (b). The temperature and strain is held constant for a period of time during which the load on the specimen decreases. The material constants for this material are E = 9850 tons/in.^{2} at 525°C and $\nu = 0.305$. Hoff's formula for a material subjected to primary creep was then derived for the stress-strain-time relation obtained from the constant-stress creep tests conducted by Johnson. This formula is

$$\dot{\epsilon}'_{yy} = 3.51 \times 10^{-7} \sigma_{yy}^{1.85} t^{-0.615} \text{ in. per in. per min.} \quad (22)$$

Following the procedure outlined in the previous section, first the rectangular structure shown in

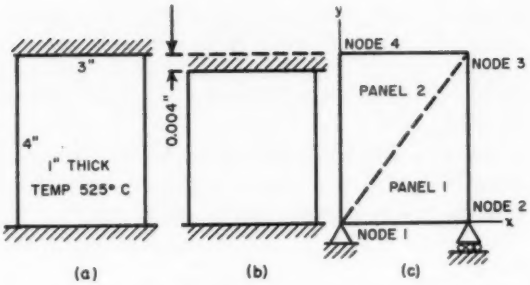

FIG. 3. COLUMN UNDER CONSTANT DEFORMATION

Fig. 3 (c) was divided into two triangular elements. The coordinates, x and y, the nodal numbers, and the panel numbers are shown in Fig. (3c).

The assembled elastic stiffness matrix, $[K_E]$, is then

$$452{,}50 \begin{bmatrix} 19.18 & -16 & 0 & -3.128 & 0 & 3.660 & -7.830 & 4.170 \\ -16 & 19.127 & -3.128 & 0 & 4.170 & -7.830 & 3.660 & 0 \\ 0 & -3.128 & 19.128 & -16 & -7.830 & 4.170 & 0 & 3.660 \\ -3.128 & 0 & -16 & 19.128 & 3.660 & 0 & 4.170 & -7.830 \\ 0 & 4.170 & -7.830 & 3.660 & 14.560 & -5.560 & 0 & -9 \\ 3.660 & -7.830 & 4.170 & 0 & -5.560 & 14.560 & -9 & 0 \\ -7.830 & 3.660 & 0 & 4.170 & 0 & -9 & 14.560 & -5.560 \\ -4.170 & 0 & 3.660 & -7.830 & -9 & 0 & -5.560 & 14.560 \end{bmatrix} \quad (23)$$

By using this stiffness matrix, the elastic analysis yields

$F_{y1} = F_{y2} = -F_{y3} = -F_{y4} = 14.78$ tons, $u_2 = u_3 = 0.00091$ in., and $u_4 = 0$.

The assembled creep stiffness matrix, $[H_E]$, for this structure is

$$\frac{t^{0.615} 10^5}{1.264\, \sigma_{yy}^{0.85}} \begin{bmatrix} 41 & -32 & 0 & -9 & 0 & -24 & 12 \\ -32 & 41 & -9 & 0 & 12 & 12 & 0 \\ 0 & -9 & 41 & -32 & -24 & 0 & 12 \\ -9 & 0 & -32 & 41 & 12 & 12 & -24 \\ 0 & 12 & -24 & 12 & 34 & 0 & -18 \\ 12 & -20 & 12 & 0 & -16 & -18 & 0 \\ -24 & 12 & 0 & 12 & 0 & 34 & -16 \\ 12 & & 12 & -24 & -18 & -16 & 34 \end{bmatrix} \quad (24)$$

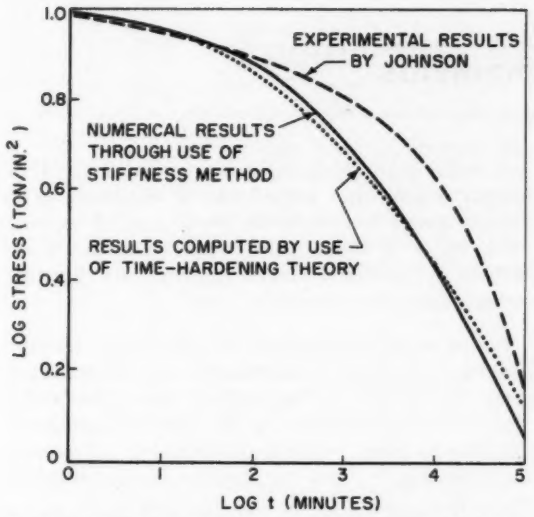

FIG. 4. STRESS RELEXATION OF CR-MO STEEL COLUMN UNDER CONSTANT UNIAXIAL COMPRESSIVE STRAIN

In order to compute the force and displacement rates, the creep stiffness matrix, $[H_R]$, is evaluated at a certain time, t, and then it is assumed to remain constant during the time increment of Δt that follows t. However, this procedure will not apply for the evaluation of $[H_R]$ at $t=0$ since $[H_R]=0$ from Eq (24); hence the creep displacement rates, $\{\dot{u}_R\}$, will be zero. Obviously, the absurdity of this result comes from Eqs (11) and (12), where the creep displacement rates, $\{\dot{u}_R\}$, and force rates, $\{\dot{F}_R\}$, are evaluated at $t=0$ instead of at t_o, where $0<t_o<\Delta t$. As an approximation, t_o will be set to $\Delta t/2$ or $t_o=\Delta t/2$ for the first time increment of t after $t=0$, while σ_{yy} in Eq (24) will be computed at $t=0$. Subsequent creep displacement and force rates will be evaluated in a similar way. The creep stiffness matrix for the first time increment was evaluated at $t_o=10$ minutes for $\Delta t=20$ minutes. The force and displacement rates thus compiled are

$$\dot{F}_{y1}=\dot{F}_{y2}=-\dot{F}_{y3}=-\dot{F}_{y4}=0.0866 \text{ ton/min},$$

$$\dot{u}^e_2=\dot{u}^e_3=-0.537\text{x}10^{-5} \text{ in/min},$$

$$u'_2=u'_3=0.882\text{x}10^{-5} \text{ in/min},$$

$$\dot{v}^e_3=-\dot{v}'_3=2.346\text{x}10^{-5} \text{ in/min}$$

$$\dot{v}^e_4=\dot{v}'_4=2.346\text{x}10^{-5} \text{ in/min}.$$

By use of these force and displacement rates, the forces and displacement after 20 minutes are calculated as

$$F_{y1}=F_{y2}=-F_{y3}=-F_{y4}=13.044 \text{ tons}.$$

With these new nodal forces, the stress in the column is recomputed; hence the creep stiffness matrix is modified. Then the new force and displacement rates are computed.

This procedure is repeated 15 times. The time intervals of the subsequent computations are not equal, but are selected in such a manner that the logarithms of the time increments are approximately equal to each other. The results of this computation in graphical form are drawn in Fig. 4. For the purpose of comparison the experimental results obtained by Johnson and the computed results by Finnie and Heller (Ref 10) are also shown in Fig. 4. The computed results are based on the time-hardening theory, which is also the basis of this derivation. The agreement between the computed results and the numerical results based on the stiffness method is good.

Conclusion

We believe that sufficient information is contained here to predict creep deformation and stress relaxation in a plane structure subjected to creep. The simple example included in this paper shows that the stiffness method is probably as accurate as analytical results based on time-hardening.

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News and Comments

The University has a new graduate program leading to the degree of Ph. D. in metallurgy. Individual research and study are to characterize this new doctoral program, in which the faculties of the other departments will participate.

The new executive officer of Aeronautical Engineering is Dr. **R. J. H. Bollard,** formerly a member of the Purdue University faculty and associate director of the Aerospace Sciences Laboratory there. Dr. Bollard's specialty is the structural problems of space vehicles.

Professor **V. B. Hammer** has been appointed executive officer of General Engineering, after serving as acting head since the summer of 1959.

Professor **T. H. Campbell** has been appointed acting executive officer of the Civil Engineering department. He relieves Professor **R. B. Van Horn,** who had headed the department for twenty years.

One hundred people attended a new short course in technical writing held at the University on September 14-16. Sponsors of the course were the Seattle chapter of the Society of Technical Writers and Publishers and the Department of Humanistic-Social Studies in the College of Engineering; co-chairmen of the committee that arranged the course were **Galen Maxfield** of the U.S. Bureau of Commercial Fisheries and **S. W. Chapman,** head of Humanistic-Social Studies.

Research on boiling-heat transfer, directed by **C. P. Costello,** Associate Professor of Mechanical Engineering, is to continue, thanks to a NSF grant of $16,800.

Roy B. Sawhill, Associate Professor of Civil Engineering, and **Joseph C. Firey,** Professor of Mechanical Engineering, are the authors of "Motor Transport Fuel Consumption Rates and Travel Time," one of three studies that make up Highway Research Board *Bulletin 276.*

F. H. Bartlett, Assistant Professor of General Engineering, has been awarded a Fulbright grant to teach at Alexandria University, Egypt, in 1962; he will lecture on marine engineering and naval architecture.

A NSF grant of $22,630 was awarded the UW nuclear reactor for design and development of a device for rapidly varying the power level of reactors. Professor **A. L. Babb,** reactor supervisor, and **W. E. Wilson,** Jr., associate reactor engineer, are directing the project.

For pioneering endeavor in highway safety, through training courses for supervisors of motor-vehicle fleets, the College of Engineering was added to the Hall of Fame established by the National Committee for Motor Fleet Supervisor Training. Professor **F. H. Rhodes,** Jr., who accepted the award, organized the College program 17 years ago.

R. H. Bogan, Associate Professor of Civil Engineering, and **Dr. D. D. Chapman** of the Aero Space Division of the Boeing Company have developed a miniature waste-treatment system adaptable to space ships, as described in a paper Bogan read at the national meeting of the Society for Industrial Biology.

MEETING OF WEST COAST CONFERENCE OF APPLIED MECHANICS

Over 200 scientists and engineers attended the 1961 West Coast Conference of Applied Mechanics held August 28-30 on the University campus. A feature of the conference was the two invited general lectures by internationally recognized authorities: Professor William Prager of Brown University, whose subject was "On the Construction of Constitutive Equations," and Professor Eric Reissner of M.I.T., whose lecture was entitled "Variational Considerations in Shell Theory." Professor Prager was formerly Walker-Ames Lecturer at the University.

The thirty-five papers on the program covered a wide range of subjects including elasticity, plasticity, thermoelasticity, fluid mechanics, elastodynamics, vibrations, plates, shells, dynamic plasticity, and crack propagation. Professor G. A. Zizicas of U.C.L.A. is chairman of the West Coast Committee of the ASME Applied Mechanics Division, and the author is chairman of the West Coast Committee of the ASCE Engineering Mechanics Division. These two organizations sponsored the meeting.

Professor B. J. Hartz,
Associate Professor of Civil Engineering

Engineering Research Dividends

Recent theses submitted for advanced degrees from the College of Engineering, University of Washington, will regularly appear in abstract form in *The Trend*. Complete copies of these theses are available for reference at the University of Washington Library. Address inquiries to the Director of the Engineering Experiment Station, University of Washington, Seattle 5.

ARDESHIR, F. S., "Preparation of Titanium Boride by the Electrolysis of Fused Salts," M.S. in Chemical Engineering, 1961.

Previous work on the electrodeposition of titanium boride from fused mixtures of potassium chloride, potassium fluotitanate, and potassium fluoborate was continued. The fused salt mixtures were electrolyzed in graphite crucibles, which were also the anodes, stainless steel being the cathode material. An inert atmosphere was maintained, and operating conditions were varied.

The effect on current efficiency of operating temperature, time of electrolysis, and current concentration in the melt was studied. By changing the operating conditions and experimental procedure, various types of cathode deposits and product compositions ranging from titanium diboride to Ti_2B_5 were obtained.

GRANT, D. W., "An Experimental Investigation of the Effect of Thickness of Backfill on Lateral Earth Pressures against a Vertical Wall," M.S. in Engineering, 1961.

An experimental investigation, on a model scale, was made into the variation in lateral forces on a vertical retaining wall due to changing thicknesses of cohesionless backfill material behind the wall. In this study the test setup was designed, the equipment was installed and calibrated, and a series of tests were run on cohesionless material to ensure that all components were functioning properly. In addition, the data from the tests were used to provide information on the magnitude of total force on the face of the wall owing to sand pressures. The backfill thickness varied from one to twelve inches. The results are presented, and the magnitudes of total lateral force and overturning moment are obtained for the range of thicknesses tested. For the greatest backfill thickness, the force determined by test and corresponding values computed by the classical theories of Coulomb and Rankine are discussed.

KRIER, A. H., "An Evaluation of the All-Way-Walk Signal Control," M.S. in Civil Engineering, 1961.

This thesis evaluates an "All-Way-Walk" signal control on the basis of a vehicular volume-stopped-time delay relationship at an urban intersection for the following signal controls or conditions: (1) Fixed signal cycle with All-Way-Walk control; (2) Fixed signal cycle with no special pedestrian control; and (3) Fixed signal cycle with a pedestrian delay or "hold" phase. A secondary part of this study was analyses of pedestrian delay, vehicle capacity, and vehicle stop cost.

Stopped-time delay was measured with a Friden calculator, and all vehicular movements were recorded. This data, summarized for 15-minute periods, was analyzed with the IBM 709 computer for various vehicular volume-stopped-time delay relationships.

The All-Way-Walk signal control was responsible for the most delay both to vehicles and pedestrians. Very little time is saved per vehicle by using a pedestrian-delay-signal phase over a system with no special pedestrian control at the particular intersection studied. The percentage of vehicle backlogged time, the vehicular volume-stopped-time delay relations, and the number of delayed vehicles can give an indication of the vehicular capacity of an intersection approach.

LIN, PAI-CHIEH, "Determination of a Gravel Equivalent for Asphaltic Pavement Mats," M.S. in Civil Engineering, 1961.

A preliminary study was made by means of a laboratory Highway Test Track to investigate the strength of asphaltic pavement slabs in terms of a gravel equivalent. The Highway Test Track consists of a loading device and a tank containing the subgrade: an A-2-4 sandy-silt soil, a crushed-rock base course, and an asphaltic concrete slab. A motor and gear box produced four different speeds from 2.25 to

7.5 fps, and the wheel load was measured with an axle-strain indicator developed for the purpose. Five pressure cells were calibrated to get the relationship of deflection and stress and then installed under the base course to measure the pavement and base-course deflection under wheel loads to get the magnitude of stresses.

The test was run on the different thicknesses of crushed-rock base course, wheel loadings, tire pressures, and speeds. Then it was run on the 2-in. thickness of asphaltic concrete slab with different wheel loads, thicknesses of base course, and tire pressures. The experimental pressure distribution beneath the base courses, which agrees with the Boussinesq equations, is confirmed by the report on Highway Research Board Proceedings, 1955, and the report of the WASHO Test.

The pressure studies led to the development of a gravel-equivalent chart showing that the asphaltic concrete slab has a much higher load-distribution power than the crushed-rock base course and that the thickness of base course can be replaced by a given thickness of asphaltic concrete slab.

SIENKIEWICZ, C. G., "Theory and Fabrication of Tunnel Diodes," M.S. in Electrical Engineering, 1961.

A discussion of tunnel-diode theory is given, including a fairly simple qualitative discussion of the current-voltage characteristic and a mathematical treatment of the tunnel current.

Three processes are described that were used in attempts to make tunnel diodes. Strip-heater techniques in open air and in a hydrogen atmosphere were used successfully. An attempt at pulse forming some tunnel junctions was not successful. After the effects of varying the time-temperature cycle and the crystal surface preparations were studied, a process was found for making good tunnel junctions.

Both arsenic- and phosphorous-doped germanium were used in making tunnel diodes. The arsenic-doped germanium was found to give better tunnel junctions.

An anomalous current-voltage characteristic observed in the course of this study is described.

TUAN, HANG-SHENG, "Frequency Scanning Antennas," M.S. in Electrical Engineering, 1961.

This paper presents the theory of 180-deg scanning for a linear array and Tchebyscheff array by frequency variation and discusses the relationships between the linear array theory and the ω-β diagram of periodic structures. Each element is excited by an equivalent slow-wave structure that is either nondispersive or dispersive. At a center frequency, the spacing is an integral multiple of guide wavelength $M\lambda_c$. This is, in fact, the $-M^{th}$ space harmonic in the so-called forbidden region, and all the other harmonics are slow. For a given frequency range, the necessary phase velocity and other parameters are determined to obtain 180-deg scanning without causing other main beams, and the example is carried out for the X-band frequency range.

WARNE, D. E., "Continued Development of an Electromechanical Analog Computer," M.S. in Mechanical Engineering, 1961.

The purpose of this project was to increase the capabilities and flexibility of an electromechanical analog-computer system capable of solving second-order linear differential equations with no driving function.

Additional computing elements were secured for integration with the system. It was decided that the device should be expanded to solve higher-order differential equations, sets of simultaneous differential equations, and also equations with a sinusoidal forcing function. In addition, improvement of the computer as a tool for use by engineering students was desired.

The development work on the system consisted of the design of a low-inertia adder bank, a sinusoidal function generator, and two geared read-out devices. Additional integrators were added to increase the order of equations handled, and a programming system was devised to make the computer easier and simpler to use.

The development work was completed, and several equations of representative types were solved. The order of magnitude of the solution error was found by comparison with solutions of known accuracy from a digital computer.

The accuracy has been demonstrated to be more than 95% when the system is properly stabilized and is operated at 1/50 of real time. The capability of the computer as an engineering tool was improved.

WINDELL, J. C., "An Electronic Reader," M.S. in Electrical Engineering, 1961.

The project was undertaken to design a device capable of automatically recognizing the letters of the Roman alphabet and the Arabic numerals. In particular, the feasibility of using simulation coding techniques in conjunction with simultaneous derivation and processing of multiple-scan information was investigated. The results of the study indicate that such techniques are feasible and could be extended to the recognition of many other sets of patterns.

GRANT FOR SANITARY ENGINEERING

A U.S. Public Health Service grant of $155,630 has been awarded the University for broadening and strengthening graduate work in sanitary engineering. The sum is to be used in a five-year program begun this fall. Professor Robert O. Sylvester is directing the project. Under the terms of the grant, the program will aid in the improvement of the professional background, instruction, and research of the sanitary engineering division of Civil Engineering. The graduate work is to be strengthened and expanded through additional courses, including sanitary biology, air pollution, and sanitary chemistry, and by adding staff members required for this course-load. Also a part of the divisional reorganization is a reduction in the teaching load of present staff members during one academic quarter, to permit course revisions and other preparatory effort.

ELECTRONIC MAPPING

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MODULATED TRAVELING-WAVE ANTENNAS

(Continued from page 8)

produce first-order variations in the power radiated by the elements, but only second-order variations in the capacitive loading to the transmission line.

The 15-wavelength experimental antenna was constructed with 10 elements per free-space wavelength, tapering from about $\lambda/8$ at the driven end to $\lambda/4$ at the far extremity, λ being the free-space wavelength. The measured pattern is shown in Fig. 4b. The highest side lobes are 15 db below the main lobe, and the beamwidth of the main lobe is about correct for the total aperture length involved. Similar end-fire patterns were obtained over a 50% frequency band.

These results are preliminary, no attention having been paid toward obtaining either the optimum amplitude distribution or the optimum loading on the transmission line. However, the results are very promising. They indicate that the driven design may overcome the major shortcomings of the traveling-wave design, yet retain most of the advantages.

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AN EXPERIMENTAL INVESTIGATION OF FLOW CHARACTERISTICS

(Continued from page 16)

of the small-scale apparatus used without seriously affecting the base pressure or the flow upstream of the probe. The slight effect of the probe indicates that a great deal of useful information on details of the flow in and near a base region may be obtained with quite small pieces of apparatus. Investigations currently under way will indicate whether such apparatus may be useful for more complex flows such as occur with the injection or removal of heated mass into a base region.

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OUTSIDE U. S. A.

A Selection from Our Foreign Exchange Publications

GEOTHERMAL ENERGY. A condensation from a U. N. Report, *Engineering Digest* (Canada), Vol. 7, No. 9, Sept., 1961.

A U.N. Conference on New Sources of Energy to be held in Rome last August was to explore further the prospects for the use of geothermal energy. Considerable developments of this sourc have been made in Iceland, New Zealand, Australia, and Russia. Prospective fields are in Japan, Hawaii, the Philippines, and South Pacific islands, Indonesia, the Congo, Tanganyika, Kenya, Ethiopia, and the west coasts of both Americas, from Canada to the tip of South America.

New Search Methods

"Following the geologist who gives the general indications, come the geophysicists and geochemists. Seismic soundings, gravimeters, magnetometers and airborne instruments play a part. Semi-conductor resistance thermometers lowered in a borehole signal back the precise temperature. Chemical content, particularly the amount of silicon present in the water or stream, gives a pretty reliable indication of the heat of the rocks it comes from. Both natural and artificial isotopes can be used to track the origins of water sources.

"All this is important because very little is really known about the real nature of the subterranean cisterns. Where they vent naturally, they are usually associated with volcanic activity, but sometimes they come from deeper sources.

"The deeper one goes into the earth's crust, the higher the temperatures. The radioactivity of the rocks spontaneously generates heat. But how does the water get in contact with the hot rocks? It has been found, for instance, that drilled wells can produce steam and gases 20 times as hot as those from the surrounding natural vents. One theory is that the man-made outlets relieve the pressures in the porous rock formations, and cold subterranean waters flow into the hot rocks and circulate as in a bathroom "geyser," coming out of the faucet piping hot.

Unlimited Supply?

"Bound up with this is the speculation as to how inexhaustible those sources may be. Some think that the supply is virtually unlimited; others that a source may give out just as an oil well spends itself. One thing appears to be true: that the ejection of hot water, steam or hot gases from any given source is constant. It does not ebb or flow or, like rivers and lakes on the surface, suffer from seasonal or cyclical variations; and that to an engineer is a great consolation. Nor does withdrawal seem to affect the temperature. New Zealand experience indicates the opposite.

"Another thing, it should be repeated, is that geothermal energy is cheap. Exploration and drilling and the installation of the surface plant costs money. Operational costs are low. For instance, an installation in California with a capacity of 12,500 kilowatts runs for 24 hours a day, and for 16 hours of that it is entirely automatic; there is no one in the power station; safety devices would switch off the plant if anything went amiss, but there has been no difficulty.

"For years, geothermal plants at Lardarello, near Pisa, have been operating with no fuss or bother. They produce two-thirds of the electricity required to run the entire Italian railway system. (In fact, they produce the equivalent of the total needs of the system, but one-third of the output is diverted to industry.) The biggest single plant there has a capacity of 100,000 kilowatts, and it has been running constantly for 10 years with a load factor of 99 per cent. The cost per kilowatt hour is 2½ mills or a fourth of an American cent, or just a little more than the British farthing.

"That is about half the most optimistic (and yet to be fulfilled) estimate of the atomic power engineers."

Only a Prelude?

"The average person daily eats two and three-quarters pounds of food, drinks four and one-half pounds of water, but breathes about twenty pounds of air. He can refuse to eat suspected food or to drink water of doubtful purity, but he must breathe such air as is available, even though it may be contaminated and a danger to health and even to life itself. Unfortunately, modern industrialized society has forced nearly half the population of American and equally large segments of the peoples of other countries to live or work in environments polluted to some degree with smoke, dust and chemical fumes from factories, processing plants, heating units, incinerators and automobile exhausts. Air pollution has thus become a social evil of widespread consequences, but it may be only a prelude to something far more disastrous. Air-borne contaminants produced by atomic and nuclear weapons are capable of blanketing the earth in the space of an hour with a deadly cloud, and enormous stocks of never-before-used war gases and bacterial agents exist which could be the parents of wave after wave of disease and death. It is high time for peoples everywhere to realize that we all live in the same ocean of air, that we draw sustenance from it, and that, if we violate the air beyond a limit, complete and irrevocable destruction will be our reward."

From Clyde Orr, Jr., in *Between Earth and Space.*